4. 3-Ports and 4-Ports Microwave Components

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References

1.0 3-Ports Microwave Components/Networks

Example of 3-Port Component - Power Divider and Combiner

- Power division and combining can be achieved with 3-Port networks.

\[ P_1 = P_2 + P_3 \]
\[ P_2 = \alpha P_1 \]
\[ P_3 = (1-\alpha)P_1 \]

\( \alpha \) is known as the division ratio
Another Example – 1-to-N Power Divider

- This is an example of 1-to-4 power divider.

General Properties of 3-Port Networks

- Like 2-Port network, n-port networks are described by their corresponding S-matrix. For a 3-port network, the S-matrix has 9 elements.

\[
\begin{bmatrix}
    s_{11} & s_{12} & s_{13} \\
    s_{21} & s_{22} & s_{23} \\
    s_{31} & s_{32} & s_{33}
\end{bmatrix}
\]

- If all ports are matched, then \( s_i = 0, \) i=1,2,3… (explain this).

\[
\begin{bmatrix}
    0 & s_{12} & s_{13} \\
    s_{23} & 0 & s_{23} \\
    s_{31} & s_{32} & 0
\end{bmatrix}
\]

- It can be shown that the S-matrix of a 3-port network cannot be matched, lossless and reciprocal at the same time. One of these characteristics has to be given up if the 3-port network is to be physically realizable, see Exercise 1.1 for the mathematical proof.
3-Port Networks (1)

- A lossless network results in Unitary S-matrix.
- When the S-matrix is non-reciprocal (s_{ij} ≠ s_{ji}), but the conditions of port match and lossless apply, the 3-port network is known as a Circulator.

Note: Here α = 1

Check that both matrices are not symmetry

\[ S_{cw\_cir} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad S_{acw\_cir} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (1.1) \]

• Circulator usually has ferrite material at the junction to cause the non-reciprocity condition.
• Read chapter 9 of [1] for the complete discussion.

3-Port Networks (2)

- A lossy 3-Port network can be reciprocal and matched at all ports. This type of network is useful as power divider, in addition it can be made to have isolation between its output ports (for instance s_{23} = s_{32} = 0).

\[ S = \begin{bmatrix} 0 & s_{12} & s_{13} \\ s_{12} & 0 & s_{23} \\ s_{13} & s_{23} & 0 \end{bmatrix} \quad (1.2a) \]

- A third type of 3-Port network, which is also used as power divider is reciprocal, lossless and matched at only 2 ports. It can be shown that its S-matrix is given by:

\[ S = \begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\theta} \end{bmatrix} \quad (1.2b) \]
Example 1.1A - Usage of circulator

In a transceiver, to separate the transmit and received signal.

Example 1.1B - Construction of Stripline Circulator

D.C. magnetic field to bias the ferrite disks

Stripline conductor

Ground planes

Ferrite disks

Metallic enclosure

Ferrite disc

Permanent magnet
Exercise 1.1

- Show that the S-matrix of a 3-port network cannot be matched, lossless and reciprocal at the same time. Hint: Use proof by contradiction, assume all three conditions are fulfilled in a 3x3 S-matrix, show that this will lead to a contradiction.

Solution...

Since the 3-port network is assumed matched and reciprocal, the matrix would be:

\[
\mathbf{S} = \begin{bmatrix}
0 & s_{12} & s_{13} \\
s_{12} & 0 & s_{23} \\
s_{13} & s_{23} & 0
\end{bmatrix}
\]  

(E1.1)

Including the unitary condition (for lossless 3-port network):

\[
\mathbf{S}^\ast \mathbf{S} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \Rightarrow \mathbf{S}^\ast = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  

(E1.2)

The diagonal elements are zero, and this matrix is symmetric.

Exercise 1.1 Cont...

Expanding (E1.2):

\[
|s_{12}|^2 + |s_{13}|^2 = 1
\]
\[
|s_{12}|^2 + |s_{23}|^2 = 1
\]
\[
|s_{13}|^2 + |s_{23}|^2 = 1
\]

(E1.3)

\[
s_{13}s_{23}^\ast = s_{12}s_{23}^\ast = s_{12}s_{23}^\ast = 0
\]

(E1.4)

To fulfill (E1.4) for arbitrary S-parameters, 2 of \(s_{12}\), \(s_{13}\) and \(s_{23}\) must be 0. Substituting this result into (E1.3), it is discovered that (E1.3) cannot be fulfilled. This leads to a contradiction, which shows that our assumption of a 3-port network with matched, reciprocal and lossless conditions is wrong.
T- Junction Power Divider

- The T-junction power divider is a simple 3-port network that can be used for power division or power combining, and can be implemented on stripline, coaxial cable, and waveguide technologies.

Equivalent electrical circuit:

Port 1 \[ Y_{in} \]

Port 2 \[ Z_1 \]

Port 3 \[ Z_2 \]

\[ Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_c} \]  \hspace{1cm} (1.3a)

We can make B to be negligible or cancel it over a band of narrow frequencies by compensation.

\[ \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_c} \]  \hspace{1cm} (1.3b)

The T-junction power divider is lossless and reciprocal. Input is matched but output is not matched. Verify it for yourself by deriving the S matrix!!

The output impedance \( Z_1 \) and \( Z_2 \) can then be selected to provide various power division ratio \( \alpha \). Quarter-wave transformers can be used to bring the output line impedance back to the desired levels.

Various Configurations of T- Junction Power Divider
Example 1.2: T- Junction Power Divider Design

- A lossless T-junction power divider has a source impedance of $Z_c = 50\,\Omega$. The impedance is matched at the input. Find the output characteristic impedance so that the input power is divided in a 2:1 ratio. Compute the reflection coefficients seen looking into the output ports.

- Implement this power divider using microstrip line on a printed circuit board.

Input power to a matched divider $P_{in} = \frac{1}{2} \frac{|V|^2}{Z_c}$  

Output powers: $P_1 = \frac{1}{2} \frac{|V|^2}{Z_1} = \frac{1}{3} P_{in}$ \hspace{1cm} $Z_1 = 3Z_c = 150\,\Omega$  

$P_2 = \frac{1}{2} \frac{|V|^2}{Z_2} = \frac{2}{3} P_{in}$ \hspace{1cm} $Z_2 = \frac{2}{3} Z_c = 75\,\Omega$

Input impedance to matched divider: $Z_{in} = 75 \parallel 150 = 50\,\Omega$

In general this is true for arbitrary power divider ratio, $\alpha$, $\frac{Z_2}{Z_1} = \frac{\alpha^2}{\alpha^2 - 1}$

Example 1.2 Cont...

Looking into the $Z_1 = 150\,\Omega$ output Tline we see:  
$50 \parallel 75 = 30\,\Omega$ \hspace{1cm} $\Gamma_1 = \frac{30 - 150}{30 + 150} = -0.666$

Looking into the $Z_2 = 75\,\Omega$ output Tline we see:  
$50 \parallel 150 = 37.5\,\Omega$ \hspace{1cm} $\Gamma_2 = \frac{37.5 - 75}{37.5 + 75} = -0.333$

Implementation on microstrip line

Port 1  
GND plane at the bottom of the PCB

Z$_{c}=50\,\Omega$

Conductor  
Z$_{1}=150\,\Omega$

Port 2

PCB dielectric  
Z$_{2}=75\,\Omega$

Port 3

Top view
Example 1.2 Cont...

- Including quarter-wave transformers and step compensations for the Tline...

<table>
<thead>
<tr>
<th>Port 1</th>
<th>Port 2</th>
<th>Port 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>50Ω</td>
<td>150Ω</td>
<td>75Ω</td>
</tr>
<tr>
<td></td>
<td>50Ω</td>
<td>61.24Ω</td>
</tr>
</tbody>
</table>

The observant reader will notice that this realization is a Narrowband power divider, by virtue of the wavelength it is only valid at the operating frequency.

T- Junction Power Divider S Matrix

- Based on the previous analysis, we can show that the S matrix for the input matched T-junction power divider is as given below (try to derive this as an exercise, using the fact that voltage on each port consists of incident and reflected components and Kirchoff’s Laws).

\[
S = \begin{bmatrix}
0 & \frac{Z_c}{Z_1} & \frac{1}{Z_1Z_2} \\
\frac{Z_c}{Z_1} & \frac{Z_c}{Z_2} & \frac{Z_c}{Z_1Z_2} \\
\frac{1}{Z_1Z_2} & \frac{Z_c}{Z_1Z_2} & \frac{Z_c}{Z_1Z_2} 
\end{bmatrix}
\]

Port 1

\[Z_{\text{in}}\]

\[Z_1, Z_2\]

Port 2

\[Z_1 \parallel Z_2 = \frac{Z_1Z_2}{Z_1 + Z_2} = Z_c\]

Port 3

\[Z_c\]

Hint:
When energize port 1, no reflection, when energize port 2 and 3, reflection exist.
Resistive Divider

- The circuit below shows a resistive divider, Port 1 is the input and Port 2 and 3 are the outputs. \( Z_1, Z_2, \) and \( Z_3 \) can be selected to give a certain power division ratio. The resistance can also be selected so that all 3 ports are matched. The circuit below can be analyzed using circuit theory.

Example 1.3 - Resistive Divider Design

- Analyze the resistive divider below, show that it is an equal split (-3 dB) divider.

The impedance \( Z \), looking into the \( Z_c/3 \) resistor followed by the output line is:

\[
Z = \frac{Z_1}{3} + \frac{Z_c}{3} = \frac{4Z_c}{3}
\]

The input impedance of the divider is:

\[
Z_{in} = \frac{Z_1}{3} + \frac{2}{3}Z_c = Z_c
\]

Since the network is symmetric for all 3 ports, all ports are matched and:

\[
S_{11} = S_{22} = S_{33} = 0
\]
Example 1.3 Cont...

Let input voltage at port 1, by voltage division rules:
\[ V = \frac{2Z_c/3}{Z_c/3 + 2Z_c/3} V_1 = \frac{2}{3} V_1 \]

Output voltage at port 2, port 3 are given by:
\[ V_2 = V_3 = \frac{Z_c}{Z_c + Z_c/3} V = \frac{3}{4} V = \frac{1}{2} V_1 \]

If we divide all port voltages by \((Z_c)^0.5\), we see that \(S_{21} = S_{12} = S_{23} = 1/2\). The network is reciprocal, so the \(S\) matrix is symmetric:

\[
\begin{bmatrix}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{bmatrix}
\]

The input and output power are:
\[ P_{in} = \frac{1}{2} \frac{V_1^2}{Z_c} \quad P_2 = P_3 = \frac{1}{2} \frac{(4V_1)^2}{8Z_c} = \frac{1}{4} \frac{V_1^2}{Z_c} = \frac{1}{4} P_{in} \]

This shows that half of the supplied power is dissipated in the resistors.

Wilkinson Power Divider (1)

- The Wilkinson power divider is a lossy 3-port network having all ports matched with isolation between the output ports.
- The Wilkinson power divider can be made to give arbitrary power division. The example shown here is the equal-split (3dB) case.

\[
S = \begin{bmatrix}
0 & j & j \\
j & 0 & 0 \\
j & 0 & 0
\end{bmatrix}
\] (1.4)

Only valid at operating frequency.
**Wilkinson Power Divider (2)**

- Despite being a lossy 3-port network, the Wilkinson divider has the property of being lossless for 'forward' wave, it is only dissipative for reflected power.

![Diagram of Wilkinson Power Divider]

**Analysis of Symmetry Wilkinson Divider (1)**

- The equal-split Wilkinson divider is a symmetrical structure, thus it can be analyzed using even and odd mode analysis.
- Any signal imposed on a pair of terminal can be split into even and odd modes components.
- For a symmetry circuit, it can be split into even and odd half circuits, the voltages at Port 1, Port 2 and Port 3 being analyzed for each modes and summed up.
- \( S \)-matrix can then be obtained for the system.

\[
V_1 = V_{even} - V_{odd} \\
V_2 = V_{even} + V_{odd} \\
V_{even} = \frac{1}{2}(V_1 + V_2) \\
V_{odd} = \frac{1}{2}(V_2 - V_1)
\]

See Chapter 7, Ref. [1] for more information.
Analysis of Symmetry Wilkinson Divider

(2)

Even mode half circuit

Odd mode half circuit

No current flows across this link, it can be disconnected.

Virtual GND

As in the 2-port case, to find the values of S-parameters, we need to energize each port individually. Suppose we energize Port 2 first.

Even excitation

Odd excitation
**Analysis of Symmetry Wilkinson Divider**

(4)

**Even circuit analysis:**
From quarter-wave transformer formula:
\[ Z_{\text{line}} = \left( \frac{\sqrt{Z_c}}{2Z_c} \right) = Z_c \]
Also
\[ \Gamma_1 = \frac{Z_c - \sqrt{Z_c}}{Z_c + \sqrt{Z_c}} = \frac{2 - \sqrt{2}}{2 + \sqrt{2}} \]
\[ V_{z2} = V_{z2}^* = V_o \quad \text{and} \quad V_{z2}^* = 0 \]
\[ V_o(t = -\frac{1}{4}) = V_0^* e^{-j\beta} + V_o^* e^{j\beta} \]
\[ = V_0^* \left( e^{-j\beta} + \Gamma_1 e^{j\beta} \right) = jV_o^* (1 - \Gamma_1) = V_o \]
\[ \Rightarrow V_o^* = \frac{V_o}{\Gamma_1} \]
\[ V_o(t = 0) = V_0^* (1 + \Gamma_1) = \frac{V_0(1 + \Gamma_1)}{\left| \Gamma_1 \right|} \]
\[ \Rightarrow V_o = -j\sqrt{2} V_o^* \]

(5)

**Odd circuit analysis:**
The quarter-wave transformer changes the shorted end into an open-circuit as looked from \( V_{2o} \). Thus:
\[ Z_{\text{line}} = Z_c \]
\[ V_{1o} = 0 \]
\[ V_{2o} = V_{2o}^* = V_o \quad \text{and} \quad V_{z2}^* = 0 \]
Finally, combining the even and odd responses to give the voltage on Port 1 and 2:

\[ V_1^+ = V_{2e}^+ + V_{2o}^+ = 2V_0 \]
\[ V_2^- = V_{2e}^- + V_{2o}^- = 0 \]
\[ V_{2o}^- = V_{2e}^- + V_{2o}^- = -j\sqrt{2}V_0 \]

Due to short and open at the line-of-symmetry (the bisection):

\[ s_{23} = s_{32} = 0 \]

By symmetry, it is obvious that similar relationship holds for voltage on Port 3, thus:

\[ s_{13} = s_{31} = -j \frac{1}{\sqrt{2}} \]
\[ s_{33} = 0 \]

Now consider the excitation source at Port 1. From the second equivalent circuit as shown below, it is obvious that \( s_{11} = 0 \).

(QED).
Microstrip Realization of Wilkinson Power Divider

- An equal power division divider:

\[
S = \frac{1}{\sqrt{2}} \begin{bmatrix}
0 & j & j \\
j & 0 & 0 \\
j & 0 & 0
\end{bmatrix}
\]

We would like to keep the separation between conductors of port 2 and conductors of port 3 as large as possible. This configuration is the best as it reduces EM field coupling between the two paths.

Example 1.4

- Design an equal-split Wilkinson power divider using microstrip line, with \(d = 1.57\text{mm}, \varepsilon_r = 4.2\) and operating frequency at 1.8GHz. The system uses microstrip line with 50\(\Omega\) characteristic impedance.
Example 1.4 - ADS Software Simulation of Wilkinson Divider (1)

For $d=1.57\text{mm}$, $\varepsilon_r = 4.2$:
- $Z_c = 50\Omega$ operating freq. $= 1.8\text{GHz}$
- $\sqrt{2}Z_c = 70.71$
- $\nu_p = 1.716\times10^8$
- $W/d = 1.048$
- $\lambda/4 = 24.0\text{mm}$

Microstrip T-junction model

To be more realistic you can also model the discontinuity at this node.

Example 1.4 - ADS Software Simulation of Wilkinson Divider (2)

$S_{22}$, indicates isolation between port 2 and port 3, should be as small as possible.

$S_{11}$, input port matching, should be as small as possible at 1.8GHz.
Unequal Power Division Wilkinson Divider

- The analysis of unequal power division Wilkinson divider has to be carried out using circuit theory as in the previous derivation, interested reader can refer to Chapter 6 of Ref. [2]. Here only the design equations are given.

\[
\begin{align*}
\frac{P_3}{P_2} &= K^2 \\
Z_2 &= Z_c \sqrt{\frac{1 + K^2}{K}} \\
Z_1 &= Z_c \sqrt{K(1 + K^2)} \\
R &= Z_c \left( K + \frac{1}{K} \right)
\end{align*}
\]

(1.5)

An Example of Equal Division Wilkinson Power Divider

- Not good, substantial coupling between two paths

(Source: [1])
Increasing the Bandwidth of Wilkinson Power Divider (1)

- Adding compensation quarter wavelength transmission line in Port 1 [5].

\[ Z_c = 50\Omega \]

Port 1

\[ Z_c = 42\Omega \]

\[ Z_c = 50\Omega \]

Port 2

\[ Z_c = 50\Omega \]

Port 3

Increasing the Bandwidth of Wilkinson Power Divider (2)

2.0 4 Ports Networks

General Properties of 4-Port Networks

(1)

- A 4-port network S-matrix contains 16 elements in a 4x4 arrangement.
- Unlike a 3-port network, a 4-port network can be lossless, reciprocal and matched at all ports simultaneously, i.e. the S-matrix has the following form (The matrix is symmetric and unitary):

\[ S = \begin{bmatrix}
0 & s_{12} & s_{13} & s_{14} \\
 s_{12} & 0 & s_{23} & s_{24} \\
 s_{13} & s_{23} & 0 & s_{34} \\
 s_{14} & s_{24} & s_{34} & 0
\end{bmatrix} \]

We have more degree of freedom, so the problem is not over-constrained, unique solution can exist.

- One way for the matrix above to satisfy unitary condition is:

\[ S = \begin{bmatrix}
0 & \alpha e^{j\theta} & \beta e^{j\delta} & 0 \\
\alpha e^{j\theta} & 0 & 0 & \beta e^{j\phi} \\
\beta e^{j\delta} & 0 & 0 & \alpha e^{j\phi} \\
0 & \beta e^{j\phi} & \alpha e^{j\phi} & 0
\end{bmatrix} \]
General Properties of 4-Port Networks (2)

• It is customary to fix $s_{12}$, $s_{13}$ and $s_{24}$ as:

$$s_{12} = s_{34} = \alpha \quad s_{13} = \beta e^{i\theta} \quad s_{24} = \beta e^{j\phi}$$

• Further application of unitary condition yields: $\theta + \phi = \pi \pm 2n\pi$

• Letting $n = 0$, there are 2 choices that is commonly used in practice.

• $\theta = \phi = \pi/2$: $\theta = 0, \phi = \pi$.

$$S = \begin{bmatrix}
0 & \alpha & j\beta & 0 \\
\alpha & 0 & 0 & j\beta \\
j\beta & 0 & 0 & \alpha \\
0 & j\beta & \alpha & 0 \\
\end{bmatrix} \quad \bar{S} = \begin{bmatrix}
0 & \alpha & \beta & 0 \\
\alpha & 0 & 0 & -\beta \\
\beta & 0 & 0 & \alpha \\
0 & -\beta & \alpha & 0 \\
\end{bmatrix}$$

These two matrices have the characteristics of a directional coupler.

• Consider the matrix of (2.2a), when an incident power wave $a_1$ is directed to port 1 (assuming all ports to be matched):

$$\begin{bmatrix}
0 & \alpha & j\beta & 0 \\
\alpha & 0 & 0 & j\beta \\
j\beta & 0 & 0 & \alpha \\
0 & j\beta & \alpha & 0 \\
\end{bmatrix} \begin{bmatrix}
a_1 \\
a_1 \\
a_1 \\
a_1 \\
\end{bmatrix} = \begin{bmatrix}
0 & \alpha a_1 \\
\alpha a_1 & 0 \\
\beta a_1 & 0 \\
\beta a_1 & 0 \\
\end{bmatrix}$$

• From the theory of S-parameters, the power delivered to port 1, port 2 and port 3 are:

$$P_1 = \frac{1}{2}|a_1|^2, \quad P_2 = \alpha^2 P_1, \quad P_3 = \beta^2 P_1$$

• From the lossless condition of the 4-port network:

$$\alpha^2 + \beta^2 = 1$$
Directional Coupler (2)

- We could repeat the previous exercise, with an incident power wave $a_2$ in port 2.

$$P_2 = \frac{1}{2} |a_2|^2 \quad P_1 = \alpha^2 P_2 \quad P_4 = \beta^2 P_2$$

- We conclude that for this particular matched 4-port network, when power is injected into a port, a portion of the power is transmitted to the opposite port while another portion is coupled to the port adjacent to the opposite port. While the adjacent port is isolated.

- This conclusion can also be arrived if we use the matrix of (2.2b), the only difference being the phase of the output voltage waves.

Directional Coupler (3)

- A 4-port network described by S-matrix of (2.2a) or (2.2b) is known as a directional coupler. The following 3 quantities are used to characterize the quality of a directional coupler.

Coupling = $C = 10 \log \frac{P_1}{P_5} = -20 \log \beta$ dB

Directivity = $D = 10 \log \frac{P_3}{P_4} = 20 \log \frac{\beta}{|S_{14}|}$ dB

Isolation = $I = 10 \log \frac{P_1}{P_2} = -20 \log |S_{14}|$ dB

(2.3)
Some Typical Directional Couplers

- Hybrid couplers are directional coupler with coupling $C = 3$ dB. This implies $\alpha = \beta = \frac{1}{\sqrt{2}}$

- Quadrature hybrid has a $90^\circ$ phase shift between port 2 and 3 when fed at port 1. It is an example of symmetrical coupler.

$$S = \begin{bmatrix}
0 & 1 & j & 0 \\
1 & 0 & 0 & j \\
j & 0 & 0 & 1 \\
0 & j & 1 & 0
\end{bmatrix}$$

- The magic T or rat-race hybrid has a $180^\circ$ phase difference between port 2 and 3 when fed at port 1, it is an example of anti-symmetrical coupler.

$$S = \begin{bmatrix}
0 & 1 & 1 & 0 \\
1 & 0 & 0 & -1 \\
1 & 0 & 0 & 1 \\
0 & -1 & 1 & 0
\end{bmatrix}$$

Branch Directional Coupler or Quadrature Hybrid (1)

- Quadrature hybrids are 3dB directional couplers with a $90^\circ$ phase difference in the outputs of the through and coupled arms. It is usually implemented in microstrip or stripline form.

$$S = \begin{bmatrix}
0 & j & 1 & 0 \\
j & 0 & 0 & 1 \\
1 & 0 & 0 & j \\
0 & 1 & j & 0
\end{bmatrix} \quad (2.4)$$
Branch Directional Coupler (2)

- The operation of the branch coupler can be analyzed using even and odd mode analysis due to its symmetrical nature. Refer to chapter 7 of [1] for the detailed analysis.

The 180° Hybrid Directional Coupler

- The 180° hybrid junction is a 4-port network with 180°/0° phase shift between the 2 output ports.
- A signal applied at port 1 will be evenly split into 2 in-phase (0°) components at port 2 and port 3, and port 4 will be isolated.
- A signal applied at port 4 will be evenly split into two components with 180° phase difference at ports 2 and 3, port 1 will be isolated.
- When operated as a combiner, input signals are applied at ports 2 and 3, the sum will be formed at port 1 while the difference will be formed at port 4.
Implementation of 180° Hybrid on Microstrip PCB

- An implementation of the 180° hybrid using microstrip line is shown below, this is commonly known as ring hybrid or rat-race. Again the analysis can be carried out using even and odd mode analysis, refer to [1] for details and more examples of implementation.

\[
S = \begin{bmatrix}
0 & 1 & 1 & 0 \\
1 & 0 & 0 & -1 \\
1 & 0 & 0 & 1 \\
0 & -1 & 1 & 0
\end{bmatrix}
\]

(2.5)

Top view

Coupled Line Directional Couplers

- When 2 unshielded Tlines are close together, power can be coupled between the lines due to interaction of EM fields of each line. For TEM or quasi-TEM mode Tlines, this can be represented in equivalent circuit for:

\[
\begin{align*}
C_{11} &= \text{Per unit length mutual capacitance} \\
L_{12} &= \text{Per unit length mutual inductance}
\end{align*}
\]
Crosstalk (1)

- This phenomenon of coupling of EM energy from one Tline to another is commonly known as crosstalk in high-speed digital circuit design.
- The amount of crosstalk is determined by $L_{12}$ and $C_{12}$.
- For instance, when an electrical pulse is send down Tline 1, electrical signal will appears at both ends of Tline 2 due to crosstalk.
- The crosstalk signals are divided into near end crosstalk (NEXT) and far end crosstalk (FEXT).

Crosstalk (2)

Note: Observe that there is a possibility for the far end crosstalk to cancel off. By proper design of the Tlines, this can be done and port 3 become isolated. This is the principle of coupled line directional coupler.
Single Stage Coupled Line DC Design (1)

- Similar with the Quadrature Hybrid and 180° Hybrid Directional Coupler, the Coupled Line Directional Coupler can be analyzed using the odd and even mode circuits concept. Please refer to Chapter 7 of [1] for more details.

- In this case, 3 impedance can be defined. (A) The characteristic impedance of each line, $Z_c$. (B) The impedance with odd mode signal is travelling along both lines, $Z_{oo}$. (C) The impedance when even mode signal is travelling along both lines, $Z_{oe}$.

- $Z_{oo}$ and $Z_{oe}$ for stripline and microstrip line configuration are widely tabulated [1].

$\lambda_0$ is the wavelength at the intended operating frequency

Coupled line design data for microstrip line, with $\varepsilon_r = 10$. Taken from Pozar, Ref. [1].

Single Stage Coupled Line DC Design (2)

$C = \text{Voltage Coupling Factor} = \frac{V_3}{V_{inc}} = \frac{R_3}{A_1}$  \hspace{1cm} (2.6a)

$Z_{oe} = Z_c \sqrt{\frac{1+C}{1-C}}$  \hspace{1cm} (2.6b)

$Z_{oo} = Z_c \sqrt{\frac{1-C}{1+C}}$  \hspace{1cm} (2.6c)
Single Stage Coupled Line DC Design

(3)

• From the previous slide, we begin the design by specifying $C$. Then use (2.6b) and (2.6c) to obtain the even and odd mode impedance. From design tables for coupled microstrip or stripline, the physical dimension such as $W$, $S$ and $d$ (dielectric) thickness can be obtained.

• Note that this design only works best at $f_0$, the intended operating frequency, it is narrowband. For a wideband directional coupler, we would need to cascade a few stages of the narrowband directional coupler.

• Finally, a good coupled line directional coupler is usually constructed using stripline, as microstrip line does not support pure TEM mode and suffers from dispersion.

The Lange Coupler

• Able to achieve a coupling $C$ of more than 3dB, not easily achieved with coupled line directional coupler. Recall than the quadrature hybrid and 180° hybrid all have coupling $C$ of 3dB only.

• Higher bandwidth than coupled line directional coupler.

• 90° phase difference between port 2 and 3.
Another Form of Lange Coupler - The Unfolded Form

```
Through
(2)

Z_c

\lambda/4

Isolated
(4)

\lambda/4

Coupled
(3)

Input
(1)

Z_c

Z_c
```

Examples of Directional Coupler (1)

- Commercial Directional Coupler with coaxial terminals
  - 880 to 2300 MHz
Examples of Directional Coupler (2)

Example of coil-type Directional Coupler from Mini-circuits Corporation
(Source: www.minicircuits.com)

Example of lumped LC Directional Coupler (narrowband).
For calculation of $L_L$, $C_{p1}$ and $C_{p2}$, see C. W. Sayre, “Complete wireless design”, 2001 McGraw Hill.

Things You Should Study Own Your Own

- Waveguide Magic-T (180° hybrid coupler).
- Tapered line 180° hybrid coupler.
- Waveguide Bethe hole coupler.
- Multi-section coupled line directional coupler.
Miscellaneous - Microwave Switches

- Can be constructed using Circulators, with the permanent magnet replaced by electromagnet.
- Mechanical-type relay.
- Semiconductor-type switch, using PIN diodes, GaAs field-effect transistor etc.

Miscellaneous - Microwave Integrated Circuit (MIC)

(Source: [1])
Miscellaneous - Example of RF/Microwave Integrated Circuit and PCB Assemblies

THE END