TDI2131 Digital Image Processing

Image Restoration & Noise Removal
Lecture 7

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Some portions of content adapted CYPee's notes, MMU. Most figures from Gonzalez/Woods
Lecture Outline

- Degradation & Restoration Process Models
- Noise Models
- Restoration in the presence of noise only (spatial filtering)
- Restoration of Periodic Noise
- Restoration in the presence of degradation & noise (frequency domain filtering)
Image Restoration

- **Image Restoration** – To improve the appearance of an image by application of a restoration process that uses a mathematical model for image degradation

- Types of degradation:
  - Blurring caused by motion or atmospheric disturbance
  - Geometric distortion caused by imperfect lenses
  - Superimposed interference pattern caused by mechanical systems
  - Noise from electronic sources
How are these images restored?
Image Restoration: The Idea

Example degraded images

Knowledge of image creation process

Develop degradation model

Develop inverse degradation process

Input image $g(x,y)$

Apply inverse degradation process

Output image $\hat{f}(x, y)$
Degradation/Restoration Process Model

- Consists of 2 parts – Degradation function & Noise function
- General model in **spatial domain**:

\[ g(x, y) = h(x, y) \ast f(x, y) + n(x, y) \]

- \( g(x,y) \): degraded image, \( h(x,y) \): degradation function, 
- \( f(x,y) \): original image, \( n(x,y) \): additive noise function
Degradation Model in Freq. Domain

- General model in frequency domain:

\[ G(u, v) = H(u, v)F(u, v) + N(u, v) \]

- \( G(x,y) \): Fourier transform of degraded image,
- \( H(x,y) \): Fourier transform of degradation function,
- \( F(x,y) \): Fourier transform of original image,
- \( N(x,y) \): Fourier transform of additive noise function

- What needs to be done??

**FIND** Degradation function and Noise model
Noise Models

• **Noise** – Any undesired information that contaminates an image

• Variety of sources:
  - Digital image acquisition process, e.g. For CCD (charged coupled device) camera, electronics signal fluctuations in detector, caused by thermal energy and light levels
  - During image transmission, e.g. Wireless network, may be corrupted by lightning or other atmospheric disturbance
Noise Models

- Assumptions of Noise models in this course:
  - Noise is independent of spatial coordinates (except for spatially periodic noise)
  - Noise is uncorrelated w.r.t the image (pixel values)

- Spatial Noise Descriptor – concern of the statistical behavior of the intensity values in the noise component of the model
  - Characterized by probability density function (PDF)
Noise Models

• Typical image noise models that can be modeled:
  – Gaussian
  – Rayleigh
  – Erlang (Gamma)
  – Exponential
  – Uniform
  – Impulse (salt-and-pepper)
Noise Models - PDFs

• Gaussian Noise

\[ p(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \]

where

- \( z \): gray scale
- \( \mu \): mean (average)
- \( \sigma \): standard deviation
Noise Models - PDFs

• Rayleigh Noise

\[
p(z) = \begin{cases} 
\frac{2}{b} (z - a) \exp\left(-\frac{(z - a)^2}{b}\right) & \text{for } z \geq a \\
0 & \text{for } z < a
\end{cases}
\]

where

mean = \( a + \sqrt{\pi b / 4} \)

variance = \( \frac{b(4 - \pi)}{4} \)
Noise Models - PDFs

• Uniform Noise

\[ p(z) = \begin{cases} 
\frac{1}{b-a} & \text{for } a \leq z \leq b \\
0 & \text{elsewhere} 
\end{cases} \]

where

mean = \( \frac{(a+b)}{2} \)

variance = \( \frac{(b-a)^2}{12} \)
Noise Models - PDFs

- **Uniform Noise**

\[
p(z) = \begin{cases} 
  P_a & \text{for } z = a \text{ (pepper)} \\
  P_b & \text{for } z = b \text{ (salt)} \\
  0 & \text{otherwise}
\end{cases}
\]

where

\[ b > a \]
Example: Noise Models

**FIGURE 5.4** Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.
Example: Noise Models

**FIGURE 5.4 (Continued)** Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.
Restoration in Presence of Noise Only – Spatial Filtering

• Consider an image degraded with only additive noise. The degradation model is further simplified as

\[ g(x, y) = f(x, y) + n(x, y) \]

in spatial domain, and

\[ G(u, v) = F(u, v) + N(u, v) \]

in frequency domain
Noise Removal with Spatial Filters

- Spatial filters can effectively remove various types of noise in digital images
- Typically operate on small neighborhoods, from 3x3 to 11x11.
- Some can be implemented as convolution masks
Mean Filters

- **Arithmetic Mean Filter**
  - Computes the average value of the corrupted image \( g(x,y) \) in area defined by \( S_{xy} \). \( S_{xy} \) represents the set of coordinates in a rectangular subimage window of size \( mn \), centred at point \( (x,y) \)
  
  \[
  \hat{f}(x, y) = \frac{1}{mn} \sum_{(s, t) \in S_{xy}} g(s, t)
  \]
  
  - The operation can be implemented using convolution mask
  
  - For random noise
Mean Filters

• Geometric Mean Filter
  - The image restored using a geometric mean filter:
    \[
    \hat{f}(x, y) = \left( \prod_{(s, t) \in S_{xy}} g(s, t) \right)^{\frac{1}{mn}}
    \]
  - For random noise: lose less detail than Arithmetic Mean

• Harmonic Mean Filter
  - The image restored using a harmonic mean filter:
    \[
    \hat{f}(x, y) = \frac{mn}{\sum_{(s, t) \in S_{xy}} \frac{1}{g(s, t)}}
    \]
  - For salt noise
Mean Filters

- **Contraharmonic Mean Filter**
  
  - The image restored with a contraharmonic mean filter:
    \[
    \hat{f}(x, y) = \frac{\sum_{(s, t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s, t) \in S_{xy}} g(s, t)^Q}
    \]

  where Q is called the order of filter

  - Positive Q: pepper
  - Negative Q: salt
  - Q=0: Arithmetic mean
  - Q=-1: Harmonic mean
Example: Mean Filters

**FIGURE 5.7**
(a) X-ray image.
(b) Image corrupted by additive Gaussian noise.
(c) Result of filtering with an arithmetic mean filter of size $3 \times 3$.
(d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)
Example: Mean Filters (Cont'd)

**FIGURE 5.8**
(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a $3 \times 3$ contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$. 
Example: Mean Filters (Cont'd)

**FIGURE 5.9** Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size $3 \times 3$ and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$. 
Order-Statistics Filters

- **Order Filter** – based on a specific type of image statistics called order statistics, sometimes known as **Order-Statistics Filter**
- **Order Statistics**: Arranges all the pixels in sequential order (smallest to largest), based on gray-level value
- The selection of the value to be replaced in the center pixel, is determined by the statistical function used (min, max, median, etc.)
Max & Min Filters

- **Minimum Filter** – select the smallest value within an ordered window of pixel values, denoted as

\[
\hat{f}(x, y) = \min_{(s, t) \in S_{xy}} \{ g(s, t) \}
\]

- Works best when the noise is primarily of the salt-type (high value)

- **Maximum Filter** – select the largest value within an ordered window of pixel values, denoted as

\[
\hat{f}(x, y) = \max_{(s, t) \in S_{xy}} \{ g(s, t) \}
\]

- Works best for pepper-type noise (low value)
Median Filters

- **Median Filter** – select the middle pixel value within an ordered window of pixel values, denoted as

  \[ \hat{f}(x, y) = \text{median}\{g(s, t)\} \]

  - Works best with salt-and-pepper noise (both high and low values)

- **Better noise remover than Averaging Filter**, which causes blurry edges and details in image, thus not effective against impulse (salt-and-pepper) noise

- Preserve line structures
Example: Median Filter

**FIGURE 5.10**
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.
(b) Result of one pass with a median filter of size $3 \times 3$.
(c) Result of processing (b) with this filter.
(d) Result of processing (c) with the same filter.
Example: Min & Max Filters

**FIGURE 5.11**
(a) Result of filtering Fig. 5.8(a) with a max filter of size $3 \times 3$. (b) Result of filtering 5.8(b) with a min filter of the same size.
Midpoint Filter

- **Midpoint Filter** – select the average of the maximum and minimum pixel values within the window, denoted as

\[
\hat{f}(x, y) = \frac{1}{2} \left[ \max_{(s, t) \in S_{xy}} \{ g(s, t) \} + \min_{(s, t) \in S_{xy}} \{ g(s, t) \} \right]
\]

- Useful for Gaussian and uniform noise
Alpha-trimmed Mean Filter

- **Alpha-trimmed Mean Filter** – select the average of the values within the window, but with some of the endpoint-ranked values excluded.

- Suppose we delete $d/2$ lowest and $d/2$ highest gray level values of $g(s,t)$ in the neighborhood $S_{xy}$. Let $g_r(s,t)$ represent the remaining $mn-d$ pixels, the remaining pixels are averaged:

$$
\hat{f}(x, y) = \frac{1}{mn-d} \sum_{(s, t) \in S_{xy}} g_r(s, t)
$$

- Useful for combination noise such as salt-and-pepper with Gaussian noise.
Example: Mean Filters

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a $5 \times 5$: (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$. 
Adaptive Filters

- Filters that consider behavior changes based on statistical characteristics of the image inside the filter region.
- Capable of superior performance, but causes increase in filter complexity
- Textbook Extra Reading:
  - Adaptive, local noise reduction filter
  - Adaptive median filter
Adaptive Filters

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Periodic Noise

• **Periodic Noise** can be effectively filtered using frequency domain techniques

• Periodic Noise: Concentrated bursts of energy in the Fourier transform, at locations corresponding to the frequencies of the periodic interference

• Approach: Use selective filters to isolate noise – **Bandreject, Bandpass, Notch filters**
Bandreject Filters

- **Bandreject Filters** – Remove noise from a certain location (or band) in the frequency domain
  - Image corrupted with additive periodic noise can be easily removed with a bandreject filter
- **Bandpass Filter** – the complement function of the Bandreject filter, performs the opposite operation.
  - Useful for isolating noise pattern for analysis
Example: Bandreject Filter

FIGURE 5.16
(a) Image corrupted by sinusoidal noise. 
(b) Spectrum of (a). 
(c) Butterworth bandreject filter (white represents 1). 
(d) Result of filtering. (Original image courtesy of NASA.)
Notch Filters

- **Notch Filter** – Rejects or passes frequencies in predefined neighborhoods about a center frequency
- Due to symmetry of the Fourier Transform, notch filters must appear in symmetric pairs about the origin in order to obtain meaningful results
- Available as **Notch Pass** and **Notch Reject** (one a complement of the other)
- Useful for removing periodic noise (horizontal, vertical, diagonal periodic lines in images) which are concentrated on one small spot in the frequency spectrum
Example: Notch (Reject) Filters

**Figure 5.18** Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.
Example: Notch Filter

FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico (note horizontal sensor scan lines). (b) Spectrum of (a). (c) Notch pass filter shown superimposed on (b). (d) Inverse Fourier transform of filtered image, showing noise pattern in the spatial domain. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)
Restoration Process Model

Degraded image $g(x,y)$

Degradation function $h(x, y)$

Noise model $n(x, y)$

Fourier Transform

Frequency Domain Filter $R(u,v)$

Restored Image $\hat{f}(x, y)$

Inverse Fourier Transform
Restoration Process

• Mathematical model:
  \[ G(u,v) = H(u,v)F(u,v) + N(u,v) \]
  where:
  \[ G(u,v) \] = Fourier transform of degraded image
  \[ H(u,v) \] = Fourier transform of degradation function
  \[ F(u,v) \] = Fourier transform of original image
  \[ N(u,v) \] = Fourier transform of additive noise function

• To obtain the restored image:
  \[ \hat{f}(x, y) = \mathfrak{F}^{-1}[\hat{F}(u, v)] = F^{-1}[R(u, v)G(u, v)] \]
  where:
  \[ \hat{f}(x, y) \] = the restored image, an approximation of
  \[ \mathfrak{F}^{-1}[\cdot] \] = the inverse Fourier transform
  \[ R(u,v) \] = the restoration (frequency domain) filter
Degradation Function?

• Question: How do we estimate the degradation function?
  – Image Observation
  – Experimentation
  – Mathematical Modeling
Inverse Filtering

• Uses the same model, with assumption of no noise. Fourier transform of degraded image:

\[ G(u, v) = H(u, v)F(u, v) + 0 \]

• Fourier transform of the original image will be:

\[ F(u,v) = \frac{G(u,v)}{H(u,v)} = G(u,v) \frac{1}{H(u,v)} \]

• To find the original image, take the inverse Fourier transform of \( F(u,v) \):

\[ f(x, y) = F^{-1}[F(u, v)] = F^{-1} \left[ \frac{G(u,v)}{H(u,v)} \right] \]

\[ = F^{-1} \left[ G(u,v) \frac{1}{H(u,v)} \right] \]
Example: Inverse Filtering

\[
H(u,v) = \begin{bmatrix}
50 & 50 & 25 \\
20 & 20 & 20 \\
20 & 35 & 22
\end{bmatrix} \quad \Rightarrow \quad \frac{1}{H(u,v)} = \begin{bmatrix}
\frac{1}{50} & \frac{1}{50} & \frac{1}{25} \\
\frac{1}{20} & \frac{1}{20} & \frac{1}{20} \\
\frac{1}{20} & \frac{1}{35} & \frac{1}{22}
\end{bmatrix}
\]

\[
\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = \frac{H(u,v)F(u,v)}{H(u,v)} + \frac{N(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}
\]
Inverse Filtering: Some Problems

- If any points in $H(u,v)$ are zero – division by zero
- Solution: Do not take zero-points of $H(u,v)$ into account
- In presence of noise:
  
  $$\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)} = \frac{H(u, v)F(u, v)}{H(u, v)} + \frac{N(u, v)}{H(u, v)} = F(u, v) + \frac{N(u, v)}{H(u, v)}$$

- As the value of $H(u,v)$ becomes very small, the second term becomes very large, and it overshadows the $F(u,v)$
- Limit the restoration to a specific radius about the origin in the spectrum – the restoration cutoff frequency
Example: Inverse Filter

Original image

Image blurred with an 11 x 11 gaussian convolution mask

Inverse filter, with cutoff frequency = 40, histogram stretched with 3% low and high clipping to show detail

Inverse filter, with cutoff frequency = 60, histogram stretched
Wiener Filter

• Also known as minimum mean-square error (MMSE) filter
• Attempt to model the error in the restored image through the use of statistical characteristics of noise
• The average error is mathematically minimized, resulting in the equation for Wiener filter:

\[
R_w(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + \left[ \frac{S_n(u, v)}{S_i(u, v)} \right]} = \frac{1}{|H(u, v)|^2 + \left[ \frac{S_n(u, v)}{S_i(u, v)} \right]}
\]

where

\( H^*(u, v) = \) complex conjugate of \( H(u, v) \)

\( S_n(u, v) = |N(u, v)|^2 = \) power spectrum of the noise

\( S_i(u, v) = |F(u, v)|^2 = \) power spectrum of the original image
Example: Wiener Filter

- Image blurred with an 11 x 11 gaussian convolution mask
- Image with gaussian noise variance = 5; mean = 0
- Inverse filter, with cutoff frequency = 80, histogram stretched with 3% low and high clipping to show detail
- Wiener filter, with cutoff frequency = 80, histogram stretched
Example: Motion Blur + Additive Noise

![Example image of motion blur and additive noise](image_url)

**Figure 5.20** (a) Image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(i) Same sequence, but with noise variance one order of magnitude less. (j)–(l) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a “curtain” of noise.
Other Frequency-Domain Restoration Filters

- Constrained Least-Squares Filter
- Geometric Mean Filter – the most general form for frequency domain restoration filters
Recommended Readings

● Digital Image Processing (3rd Edition), Gonzalez & Woods,
  ● Chapter 4: Image Restoration
    • 5.1 – 5.4, 5.6 – 5.10 (Week 7)
  ● Chapter 9: Morphological Image Processing
    • 9.1 – 9.4 (Week 8)