Noise Figure Calculation Exercise Using Microwave CAD Software

1.0 Derivation of Noise Figure Expression for Computer Simulation

A simple noise analysis of the resistive network is carried out.

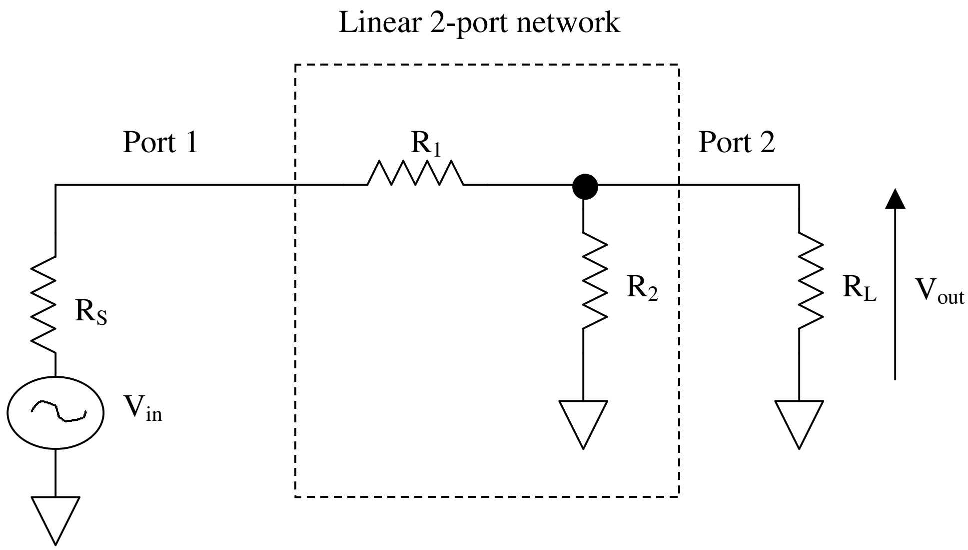

Figure 1 – A two-port resistive network.

The output voltage at Port 2 is given by:

$$V_{out} = \frac{R_L // R_2}{R_L // R_2 + R_1 + R_S} V_{in} = A \cdot V_{in} \tag{1}$$

In equation (1) A is the effective voltage gain. Deactivating the source voltage V_{in} and considering the thermal noise from the resistors, the equivalent circuit of Figure 2 is obtained.

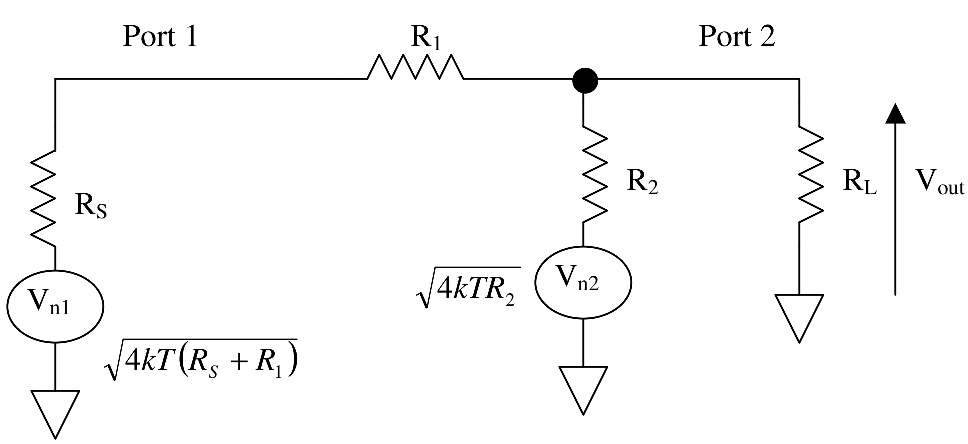

Figure 2 – Equivalent circuit with thermal noise source, signal source deactivated.

The noise sources V_{n1} and V_{n2} are white noise, and the Power Spectral Density (PSD) S_{n1} and S_{n2} which are assumed constant with respect to frequency.

$$S_{n1}(f) = 4kT(R_S + R_1) \tag{2a}$$

$$S_{n2}(f) = 4kTR_2 \tag{2b}$$

Assuming a bandwidth of Δf. Using the fact that $\overline{V_{n1}}^2 = \int S_{n1}(f)df$ and $\overline{V_{n2}}^2 = \int S_{n2}(f)df$, the contribution of the noise sources individually at the output node is given as follows:

$$V_{out(n1)} = \sqrt{\overline{V_{out(n1)}}^2} = \frac{R_2 // R_L}{R_2 // R_L + R_1 + R_S}\sqrt{4kT(R_S + R_1)\Delta f} \tag{3a}$$

$$V_{out(n2)} = \sqrt{\overline{V_{out(n2)}}^2} = \frac{(R_1 + R_S)// R_L}{(R_1 + R_S)// R_L + R_2}\sqrt{4kTR_2\Delta f} \tag{3b}$$

The random process due to V_{n1} and V_{n2} can be combined using **Superposition Theorem** for linear circuit. Since noise sources V_{n1} and V_{n2} are uncorrelated, the total PSD at port 2 is (See [1], Section11.4):

$$S(f) = S_{n1}(f) + S_{n2}(f) \tag{4a}$$

Or

$$\overline{V_{out(noise)}}^2 = \overline{V_{out(n1)}}^2 + \overline{V_{out(n2)}}^2 \tag{4b}$$

This implies:

$$V_{out(noise)} = \sqrt{\overline{V_{out(noise)}}^2} = \left[\overline{V_{out(n1)}}^2 + \overline{V_{out(n2)}}^2\right]^{\frac{1}{2}}$$

$$= \left\{\left[\frac{R_2 // R_L}{R_2 // R_L + R_1 + R_S}\sqrt{4kT(R_S + R_1)\Delta f}\right]^2 + \left[\frac{(R_1 + R_S)// R_L}{(R_1 + R_S)// R_L + R_2}\sqrt{4kTR_2\Delta f}\right]^2\right\}^{\frac{1}{2}} \tag{5}$$

Not that in computing the total noise voltage, the contribution from the termination at port 2, e.g. R_L is not considered. The signal-to-noise ratio at the input, SNR_{in} is given by:

$$SNR_{in} = \frac{V_{in}^2}{4kTR_S} \tag{6}$$

The signal-to-noise ratio at the output, SNR_{out} is given by:

$$SNR_{out} = \frac{(|A| \cdot V_{in})^2}{\overline{V_{out(noise)}}^2} \tag{7}$$

Thus the noise factor of the network, from (6) and (7) is:

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{\overline{V_{out(noise)}}^{2}}{|A|^2 \cdot 4kTR_s} \qquad (8)$$

The noise figure (F) in dB is then given by

$$F = 10 \cdot \log_{10}\left[\frac{SNR_{in}}{SNR_{out}}\right] \qquad (9)$$

Equations (8) and (9) are usually employed by computer simulation software to determine the noise figure F of a linear circuit.

2.0 Numerical Example

Let R_S = RL = 50, R_1 = 50 and R_2 = 100. T = 25° C or 298K, k = 1.380×10^{-23} JK^{-1}. Bandwidth Δf = 1Hz (spot noise calculation).

Then:

$$V_{out(n1)} = 0.25 \cdot 1.28256 \times 10^{-9} = 320.64\,pV$$

$$V_{out(n2)} = 0.25 \cdot 1.28256 \times 10^{-9} = 320.64\,pV$$

$$A = 0.25$$

$$NF = \frac{2 \times (320.64\,pV)^2}{0.25^2 \cdot (906.91\,pV)^2} \cong 4.00$$

$$F = 10\log_{10}(4.00) = 6.02$$

3.0 ADS Simulation of Small-signal Noise

In ADS the noise simulation is typically performed during AC or S-parameter simulation. Therefore it will be convenient if the noise factor NF can be expressed in terms of the S-parameters.

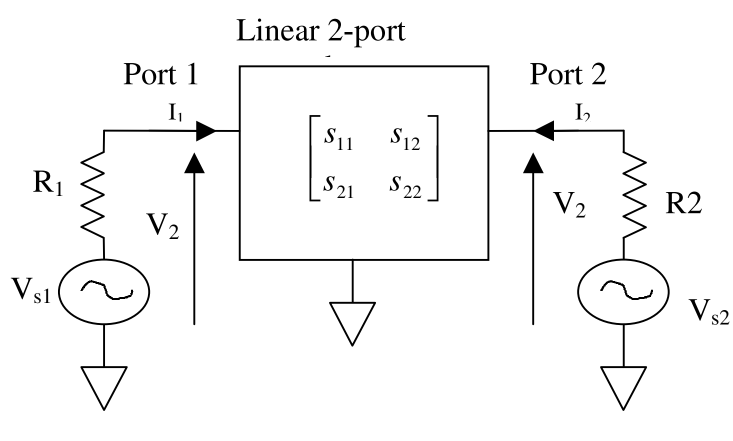

Figure 3 – Two-port network.

Consider a two-port network as shown in Figure 3, the normalized incident and reflected voltage waves can be defined according to Chapter 1 of [3]:

$$a_i = \frac{V_i + R_i I_i}{2\sqrt{R_i}} = \frac{V_{si}}{2\sqrt{R_i}} \tag{10a}$$

$$b_i = \frac{V_i - R_i I_i}{2\sqrt{R_i}} = \frac{2V_i - V_{si}}{2\sqrt{R_i}} \tag{10b}$$

Where i = 1 or 2. When measuring parameter s_{ij}, the source V_{si} would be deactivated (e.g. shorted). For s_{21}:

$$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

Generator $V_{s2} = 0$. Hence using the definition (10a) and (10b):

$$s_{21} = \frac{2V_2 / 2\sqrt{R_2}}{V_{s1} / 2\sqrt{R_1}} = 2\left(\frac{V_2}{V_{s1}}\right)\left(\sqrt{\frac{R_1}{R_2}}\right)$$

$$|s_{21}|^2 = 4\left|\frac{V_2}{V_1}\right|^2 \frac{R_S}{R_L} = 4|A|^2 \frac{R_S}{R_L} \tag{11}$$

Since $R_1 = R_S$, $R_2 = R_L$, $V_1=V_{in}$ and $V_2 = V_{out}$. Now consider equation (8) again,

$$NF = \frac{\overline{V_{out(noise)}^{\,2}}}{|A|^2 \cdot 4kTR_s} = \frac{\dfrac{\overline{V_{n(2-port)}^{\,2}} + 4kTR_S|A|^2}{R_L}}{\dfrac{\left(4|A|^2 R_S\right)}{R_L} \cdot kT}$$

Here the total mean-square noise voltage at the output is divided into contribution from the source resistance R_S and the noise sources within the two-port network. $V_{n(2\text{-port})}$ is the noise voltage at the output due to noise sources within the two-port network. Now using (11) this can be written as:

$$NF = \frac{\dfrac{\overline{V_n^{\,2}}}{R_L} + kT|s_{21}|^2}{kT|s_{21}|^2} \tag{12}$$

Equation (12) is the expression used by ADS software for noise figure computation when noise calculation is enabled during S-parameter simulation. It can be easily extended to multi-port network as illustrated in the online documentation in ADS.

4.0 ADS Simulation Example

Here the numerical example of Section 3.0 is repeated in ADS (version 2003C is used). The schematic and data display are shown in Figure 4 and Figure 5 respectively. The S-parameter is performed at a single frequency of 1.0GHz.

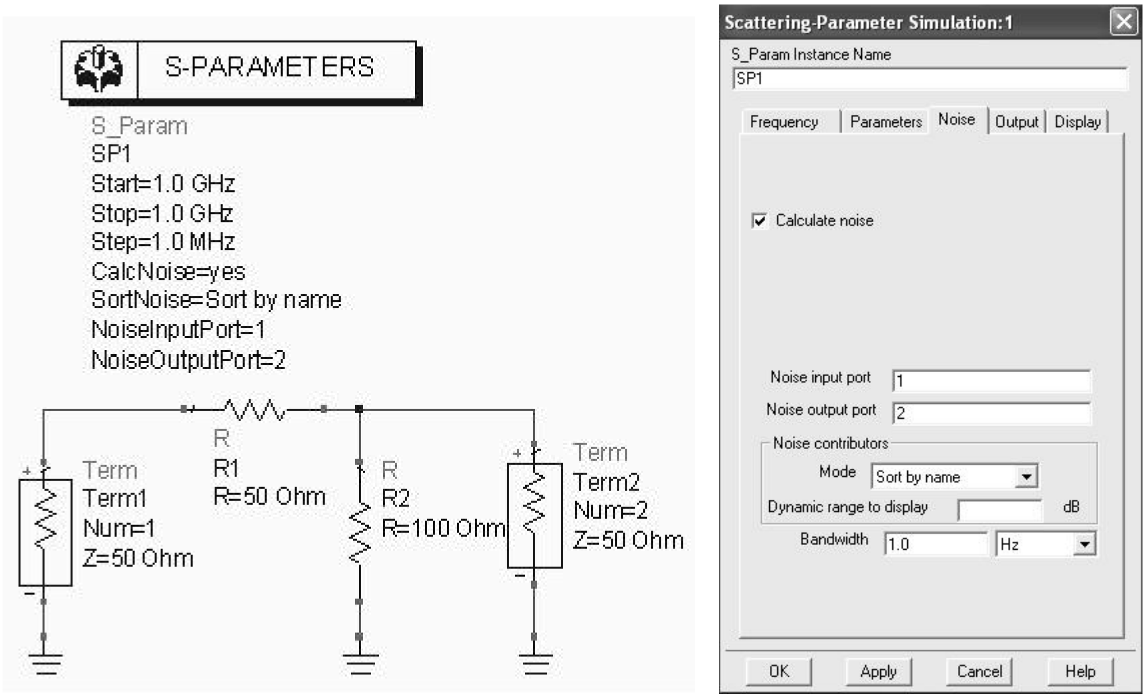

Figure 4 – The schematic and "Noise" tab setting in the S-parameter control.

freq	NFmin	Rn	nf(2)	Sopt
1.000GHz	5.808	77.108	6.111	0.268 / 0.000

index	port1.NC.name	port2.NC.name	port1.NC.vnc	port2.NC.vnc
0	_total	_total	376.2pV	392.9pV
1	R1	R1	340.3pV	226.8pV
2	R2	R2	160.4pV	320.8pV

freq	S(2,1)
1.000GHz	0.500 / 0.000

Figure 5 – The data display.

Here we are only interested in the noise figure with output being taken at port 2. So we select **nf(2)** to be shown in the list. Note that the software also calculates the minimum noise figure $\mathbf{NF_{min}}$ and the corresponding optimum source reflection coefficient $\mathbf{S_{opt}}$. See [4] on how to find S_{opt} and NF_{min}. In Figure 5, the contribution of the individual noise source at the output port is also displayed. We are only interested in the listing under **port2.NC.name** and **port2.NC.vnc** for output taken at port 2. Compare the values with those obtained in Section 3.0.

References

1. B. P. Lathi, "Modern digital and analog communication systems", 3rd edition 1998, Oxford University Press.
2. B. Razavi, "RF microelectronics", 1998, Prentice-Hall Inc.
3. G. D. Vendelin, A. M. Pavio, U. L. Rohde, "Microwave circuit design using linear and nonlinear techniques", 1990, John Wiley & Sons.
4. R. Ludwig, P. Bretchko, "RF circuit design – theory and applications", 2000, Prentice-Hall Inc.