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# 8A – Small-Signal Amplifier Design: Design for Maximum Power Gain and Design for Fixed Transducer Power Gain

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## References

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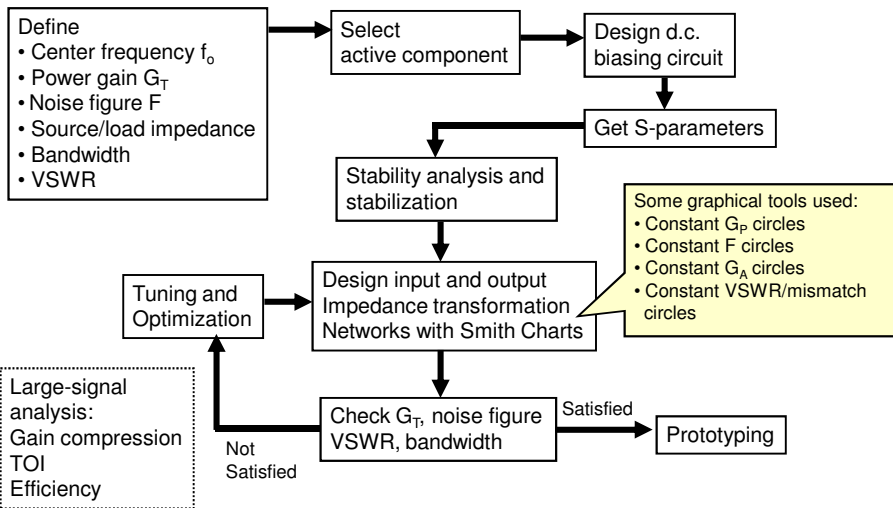
- [1]\* D.M. Pozar, "Microwave engineering", 3rd Edition, 2005 John-Wiley & Sons.
- [2] R.E. Collin, "Foundations for microwave engineering", 2nd Edition, 1992 McGraw-Hill.
- [3] R. Ludwig, P. Bretchko, "RF circuit design - theory and applications", 2000 Prentice-Hall.
- [4]\* G. Gonzalez, "Microwave transistor amplifiers - analysis and design", 2nd Edition 1997, Prentice-Hall.
- [5] Gilmore R., Besser L., "Practical RF circuit design for modern wireless systems", Vol. 1 & 2, 2003, Artech House.

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## Basic Single-Stage Small-Signal Amplifier Design Flow



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## Linear/Quasi-Linear Amplifier Types

Amplifier Types	Required $\Gamma_s$	Required $\Gamma_L$	Graphical tools
Maximum Unilateral $G_T$	$S_{11}^*$	$S_{22}^*$	None
Maximum $G_{T(max)}$	$\Gamma_{sm}$	$\Gamma_{Lm}$	None
Fixed $G_T$ with user defined F and input mismatch	Tunable	Tunable	<ul style="list-style-type: none"> <li>• Constant <math>G_p</math> circle</li> <li>• Constant F circle</li> <li>• Constant VSWR/Mismatch circle</li> <li>• Constant <math>G_A</math> circle</li> </ul>
Lowest noise	$\Gamma_m$	Tunable (typically = $\Gamma_2^*$ )	Constant F circle
Maximum Output Power	$\Gamma_{sm}$	$\Gamma_{OL}$	Constant $P_L$ contour

Good for lower frequency,  $f_0 < 500\text{MHz}$ . Not discussed here, see [1], [4] and [5]

Discussed in High Power Circuits

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# 1.0 Maximum $G_T$ Amplifier - Simultaneous Conjugate Match

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## Simultaneous Conjugate Match in Amplifier (1)

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- To maximize power transfer in amplifier, both input and output ports of the amplifier have to be conjugately matched.
- When both input and output impedance of the amplifier are conjugately matched to the source and load impedance, the amplifier is called **Simultaneous Conjugately Match (SCM)**.
- It is not easy to achieve SCM, as  $\Gamma_L$  affects  $\Gamma_1$ , and  $\Gamma_s$  affects  $\Gamma_2$ . Only under special condition can SCM occurs.

$$\Gamma_1 = \frac{S_{11} - D\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_2 = \frac{S_{22} - D\Gamma_s}{1 - S_{11}\Gamma_s}$$

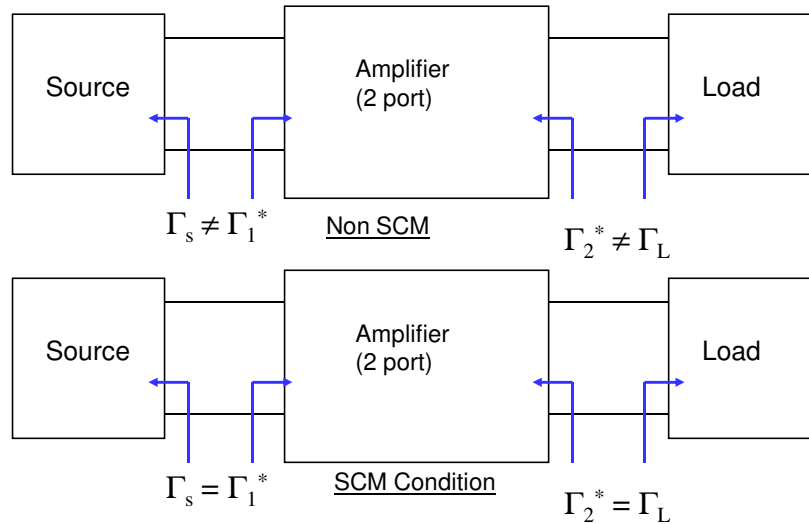
- Under SCM,  $G_p = G_A = G_T$ , and the gain obtained from the amplifier is the **maximum,  $G_{T(\max)}$** .
- Most of the time the condition for SCM is not fulfilled and we can only optimized the power gain of the amplifier while  $G_{T(\max)}$  is not attainable.

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## Simultaneous Conjugate Match in Amplifier (2)



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## Load and Source Impedance for Simultaneous Conjugate Match (1)

- The input and output reflection coefficient of an amplifier are given by:
 
$$\Gamma_1 = \frac{S_{11} - D\Gamma_L}{1 - S_{22}\Gamma_L} = \text{Input Reflection Coeff.} \quad \Gamma_2 = \frac{S_{22} - D\Gamma_s}{1 - S_{11}\Gamma_s} = \text{Output Reflection Coeff.}$$
- When we want the input and output port of the amplifier to be conjugately matched to source and load impedance,  $\Gamma_s = \Gamma_1^*$  and  $\Gamma_L = \Gamma_2^*$ .
- Thus we need to solve the following simultaneous equations:
 
$$\Gamma_s^* = \Gamma_1 = \frac{S_{11} - D\Gamma_L}{1 - S_{22}\Gamma_L} \quad \Gamma_L^* = \Gamma_2 = \frac{S_{22} - D\Gamma_s}{1 - S_{11}\Gamma_s}$$
- This can only happen for a particular set of source and load impedance (as  $\Gamma_s$  affects  $\Gamma_2$  and  $\Gamma_L$  affects  $\Gamma_1$ ).

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## Load and Source Impedance for Simultaneous Conjugate Match (2)

- Solving both equations simultaneously results in (1.1a) and (1.1b), Example 9-10 of Ludwig, Bretchko [3] provide the details.

$$\Gamma_{Sm} = \frac{1}{2B_1} \left( A_1 - \left( A_1^2 - 4|B_1|^2 \right)^{\frac{1}{2}} \right)$$

$$A_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |D|^2$$

$$B_1 = S_{11} - DS_{22}^*$$

**(1.1a)**

$$\Gamma_{Lm} = \frac{1}{2B_2} \left( A_2 - \left( A_2^2 - 4|B_2|^2 \right)^{\frac{1}{2}} \right)$$

$$A_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |D|^2$$

$$B_2 = S_{22} - DS_{11}^*$$

**(1.1b)**

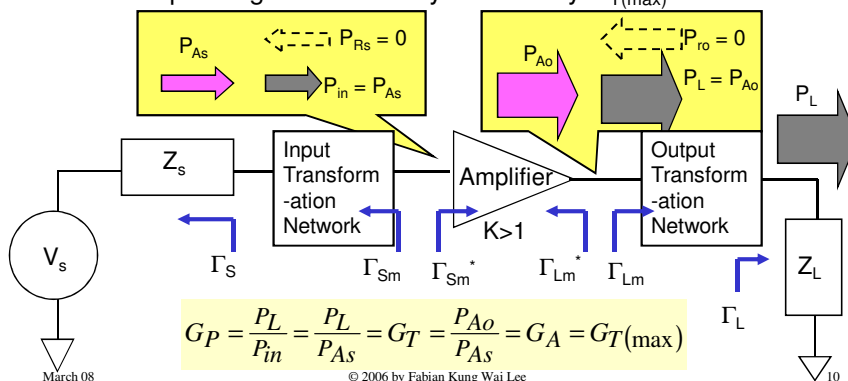
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## Power Gain During Simultaneous Conjugate Match (1)

- When we connect source and load impedance having  $\Gamma_{Sm}$  and  $\Gamma_{Lm}$  to an amplifier,  $P_{in} = P_{As}$ ,  $P_L = P_{Ao}$ .
- As a result  $G_P = G_T = G_A$ .
- Also note that transducer power gain  $G_T$  under this condition is the maximum power gain! Commonly denoted by  $G_{T(max)}$ .



## Power Gains During Simultaneous Conjugate Match (2)

- Substituting (1.1a) and (1.1b) into the expression for  $G_P$  will give us  $G_{T(\max)}$ .

$$G_P = G_T = G_A = G_{T(\max)}$$

$$G_P = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2 (1 - |\Gamma_1|^2)}$$

$\Gamma_{Lm} \rightarrow \Gamma_1 = \frac{S_{11} - D\Gamma_L}{1 - S_{22}\Gamma_L}$

$$G_{T(\max)} = \frac{|S_{21}|}{|S_{12}|} \left( K - \sqrt{K^2 - 1} \right) \quad (1.2a)$$

Roulette Stability Factor  $\rightarrow$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |D|^2}{2|S_{12}S_{21}|} \quad (1.2b)$$

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## Summary for SCM Amplifier Design

- SCM to obtain maximum power gain, can only be applied when the amplifier is unconditionally stable with input and output port conjugately matched.
- And when this happens, the maximum power gain that can be obtained is:

$$G_{T(\max)} = \frac{|S_{21}|}{|S_{12}|} \left( K - \sqrt{K^2 - 1} \right) \quad G_{MS} = \frac{|S_{21}|}{|S_{12}|} \quad (1.3)$$

- The limit for  $G_{T(\max)}$  is  $G_{MS}$ . It is the largest maximum stable power gain that can be achieved for an amplifier.
- From the above relations for  $G_{T(\max)}$ , it is only real if  $K \geq 1$ . **Thus an amplifier can only be simultaneously conjugate matched when it is unconditionally stable (pass K factor test).**
- If the amplifier is only conditionally stable, then SCM or maximum power gain cannot be achieved. The required values for  $\Gamma_{Lm}$  and  $\Gamma_{Sm}$  will fall outside the unit disc of the Smith Chart. We can only optimize  $G_T$  up to a certain point.

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## Example 1.1 - SCM Amplifier Design

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- A BJT with  $I_C=10\text{mA}$  and  $V_{CE}=6\text{V}$  is operated at  $2.4\text{GHz}$ . The corresponding S-parameters are  $S_{11}=0.3\angle 30^\circ$ ,  $S_{12}=0.2\angle -60^\circ$ ,  $S_{21}=2.5\angle -80^\circ$ ,  $S_{22}=0.2\angle -15^\circ$ . Determine whether the transistor is unconditionally stable and find the values for  $\Gamma_s$  and  $\Gamma_L$  that provide maximum power gain. Sketch the schematics of the system (use a box to represent the amplifier).

## Solution to Example 1.1

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$$S_{11} = 0.260 + j0.150$$

$$S_{21} = 0.434 - j2.462$$

$$S_{12} = 0.100 - j0.173$$

$$S_{22} = 0.193 - j0.052$$

Stability Test:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |D|^2}{2|S_{12}S_{21}|} = 1.178 \quad |D| = |S_{11}S_{22} - S_{12}S_{21}| = 0.555$$

Since  $K > 1$  and  $|D| < 1$ , the amplifier is unconditionally stable. We can maximize the power gain  $G_p$  by carrying out simultaneous conjugate match design.

## Solution to Example 1.1 Cont...

Use (1.1a) to find

$$\Gamma_{Lm} = \frac{1}{2B_2} \left( A_2 - \sqrt{A_2^2 - 4|B_2|^2} \right) = 0.044 + j0.116$$

Use (1.1b) to find

$$\Gamma_{sm} = \frac{1}{2B_1} \left( A_1 - \sqrt{A_1^2 - 4|B_1|^2} \right) = 0.281 - j0.091$$

Convert to impedance:

$$Z_{Lm} = Z_o \frac{1+\Gamma_{Lm}}{1-\Gamma_{Lm}} = 53.085 + j12.508$$

$$Z_{sm} = Z_o \frac{1+\Gamma_{sm}}{1-\Gamma_{sm}} = 86.889 - j17.325$$

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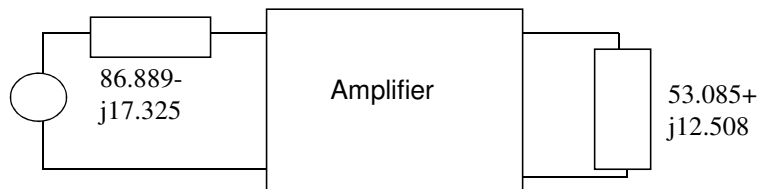
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## Solution to Example 1.1 Cont...

Finding maximum power gain:

$$G_{p(\max)} = \frac{|S_{21}|}{|S_{12}|} \left( K - \sqrt{K^2 - 1} \right) = 6.942$$



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## Appendix 1

# Derivation of Source and Load Impedance for Simultaneous Conjugate Match and Constant Power Gain Circle

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## The Condition for Maximum $G_P$ (1)

- We start by looking at the expression for power gain  $G_P$ .

$$\Gamma_1 = \frac{S_{11} - D\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_2 = \frac{S_{22} - D\Gamma_s}{1 - S_{11}\Gamma_s}$$

Even though the method presented here is much longer, it gives more insight into the characteristics of the power gain  $G_P$ .

$$|G_P| = \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2(1 - |\Gamma_1|^2)} = \frac{(1 - |\Gamma_L|^2)|S_{21}|^2}{|1 - S_{22}\Gamma_L|^2 - |S_{11} - D\Gamma_L|^2} = g_2|S_{21}|^2$$

$$g_2 = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2 - |S_{11} - D\Gamma_L|^2}$$

We call this the normalized power gain. Notice that it is dependent on  $\Gamma_L$ .

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## The Condition for Maximum $G_p$ (2)

- If we could maximize  $|g_2|$ , then  $G_p$  can be maximized.
- Using

$$\Gamma_L = U_2 + jV_2 \quad C_2 = S_{22} - DS_{11}^*$$

$$g_2 = \frac{1 - |\Gamma_L|^2}{1 - |S_{11}|^2 + |\Gamma_L|^2 (|S_{22}|^2 - |D|^2) - 2 \operatorname{Re}(\Gamma_L C_2)}$$

Because  $|S_{21}|$  is fixed, it is an important parameter in amplifier design. It sets the limit of power gain.

- Using
- We can rewrite the above expression as (See Chapter 1 of [3]):

$$U_2^2 + V_2^2 + \frac{2U_2 g_2 \operatorname{Re}(C_2)}{-1 - |S_{22}|^2 g_2 + |D|^2 g_2} - \frac{2V_2 g_2 \operatorname{Im}(C_2)}{-1 - |S_{22}|^2 g_2 + |D|^2 g_2} = \frac{-1 + g_2 - |S_{11}|^2 g_2}{-1 - |S_{22}|^2 g_2 + |D|^2 g_2}$$

## The Condition for Maximum $G_p$ (3)

- This can be further expanded into (see extra notes) a circle equation in the complex plane:

$$\left[ U_2 + \frac{g_2 \operatorname{Re}(C_2)}{-1 - |S_{22}|^2 g_2 + |D|^2 g_2} \right]^2 + \left[ V_2 - \frac{g_2 \operatorname{Im}(C_2)}{-1 - |S_{22}|^2 g_2 + |D|^2 g_2} \right]^2 = \rho_{2c}^2 \quad (\text{A.1})$$

- Where

Coordinate for center of a circle

The radius of a circle

$$\rho_{2c}^2 = \frac{-1 + g_2 - |S_{11}|^2 g_2}{-1 - |S_{22}|^2 g_2 + |D|^2 g_2} + \frac{g_2^2 |S_{22} - DS_{11}^*|^2}{(-1 - |S_{22}|^2 g_2 + |D|^2 g_2)^2} \quad (\text{A.2})$$

## The Condition for Maximum $G_p$ (4)

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- $\rho_{2c}^2$  can be further simplified using the identity

$$\operatorname{Re}^2(C_2) + \operatorname{Im}^2(C_2) = |S_{22} - DS_{11}^*|^2 = |S_{12}S_{21}|^2 + (1 - |S_{11}|)^2 (|S_{22}|^2 - |D|^2)$$

- To

$$\rho_{2c}^2 = \frac{1 - 2K|S_{12}S_{21}|g_2 + |S_{12}S_{21}|^2 g_2^2}{(-1 - |S_{22}|^2 g_2 + |D|^2 g_2)^2} \quad (\text{A.3})$$

- Where

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |D|^2}{2|S_{12}S_{21}|} \quad (\text{A.4})$$

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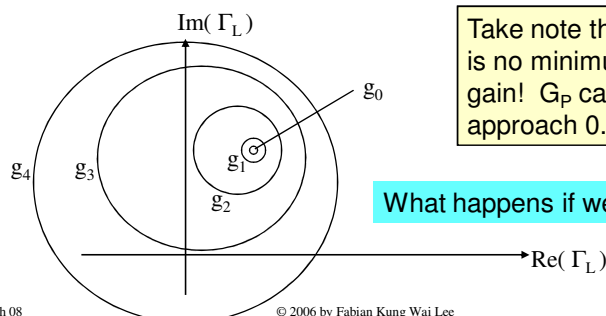
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## The Condition for Maximum $G_p$ (5)

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- Expression (A.1) again represents a circle in the complex  $\Gamma_L = U_2 + jV_2$  plane.
- For each value of  $g_2$ , a circle can be drawn. Thus each circle represents the contour of constant power gain  $G_p$  for a given load  $Z_L$ . When  $\rho_{2c}^2 = 0$ , the power gain is the maximum.



Take note that there is no minimum power gain!  $G_p$  can only approach 0.

What happens if we set  $g_2 > g_0$  ?

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## The Condition for Maximum $G_p$ (6)

- From (A.3), when  $\rho_{2c}^2=0$ :

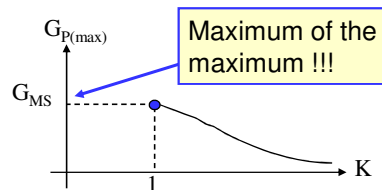
$$g_{2(\max)} = \frac{1}{|S_{12}S_{21}|} \left( K - \sqrt{K^2 - 1} \right)$$

- Since  $g_{2(\max)}$  only exist if  $K > 1$ , this implies maximum gain only exist when the amplifier is unconditionally stable.

- At  $K=1$ ,

$$g_{2(\max)} = \frac{1}{|S_{12}S_{21}|}$$

$$G_{P(\max)} = \frac{1}{|S_{12}S_{21}|} \cdot |S_{21}|^2 = \frac{|S_{21}|}{|S_{12}|}$$



- And this power gain is known as **Maximum Stable Gain,  $G_{MS}$** . This is the maximum power gain that is achievable !!! It is often specified in datasheet of an amplifier or amplification device and gives the theoretical maximum gain achievable.

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## Corresponding Load Impedance for Maximum Power Gain

- From (A.1) and putting  $\rho_{2c}^2=0$ :

$$\left[ U_{2m} + \frac{g_2 \operatorname{Re}(C_2)}{-1 - |S_{22}|^2 g_2 + |D|^2 g_2} \right]^2 + \left[ V_{2m} - \frac{g_2 \operatorname{Im}(C_2)}{-1 - |S_{22}|^2 g_2 + |D|^2 g_2} \right]^2 = 0$$

$$\Rightarrow U_{2m} + \frac{g_2 \operatorname{Re}(C_2)}{-1 - |S_{22}|^2 g_2 + |D|^2 g_2} = 0 \quad (\text{A.5a})$$

$$\Rightarrow V_{2m} - \frac{g_2 \operatorname{Im}(C_2)}{-1 - |S_{22}|^2 g_2 + |D|^2 g_2} = 0 \quad (\text{A.5b})$$

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## Load Impedance for Maximum Power Gain

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- By expanding (A.5a) and (A.5b) using  $g_{2(\max)}$ , we can show that:

- Or 
$$\Gamma_{Lm} = U_{2m} + jV_{2m}$$

$$\Gamma_{Lm} = \frac{1}{2B_2} \left( A_2 - \left( A_2^2 - 4|B_2|^2 \right)^{\frac{1}{2}} \right)$$

$$A_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |D|^2 \quad (\text{A.6a})$$

$$B_2 = S_{22} - DS_{11}^*$$

- The proof is extremely tedious and will not be shown here. You can refer to the extra hand-written notes for the proof.

## Source Impedance for Maximum Power Gain

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- Using 
$$\Gamma_1 = \frac{S_{11} - D\Gamma_L}{1 - S_{22}\Gamma_L}$$

- We can obtain the corresponding  $\Gamma_1$ .
- And requiring that  $\Gamma_S = \Gamma_1^*$ , the source reflection coefficient can be obtained.

$$\Gamma_{Sm} = \frac{1}{2B_1} \left( A_1 - \left( A_1^2 - 4|B_1|^2 \right)^{\frac{1}{2}} \right)$$

$$A_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |D|^2 \quad (\text{A.6b})$$

$$B_1 = S_{11} - DS_{22}^*$$

## 2.0 Design for Fixed Transducer Power Gain $G_T$ – The Constant $G_P$ Circle Approach

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### Constant Power Gain Circles (1)

- From the Dependency Diagram of previous chapter we see that  $G_P$  only depends on  $\Gamma_L$ . Indeed it is possible to determine the range of  $\Gamma_L$  which would give a fixed  $G_P$ , and this range actually falls on the locus of a circle on Smith Chart.
- From the equation (A.1) of Appendix 1, the radius and center of the Constant Power Gain Circle are given by:

$$C_2 = s_{22} - Ds_{11}^* \quad (2.1a)$$

$$g_2 = \frac{G_P}{|s_{21}|^2} \quad (2.1b)$$

Normalized  $G_P$

Center of circle

$$T_{GP} = \frac{-g_2 \operatorname{Re}(C_2)}{-1 - |s_{22}|^2 g_2 + |D|^2 g_2} + j \frac{g_2 \operatorname{Im}(C_2)}{-1 - |s_{22}|^2 g_2 + |D|^2 g_2} \quad (2.1c)$$

Radius of circle

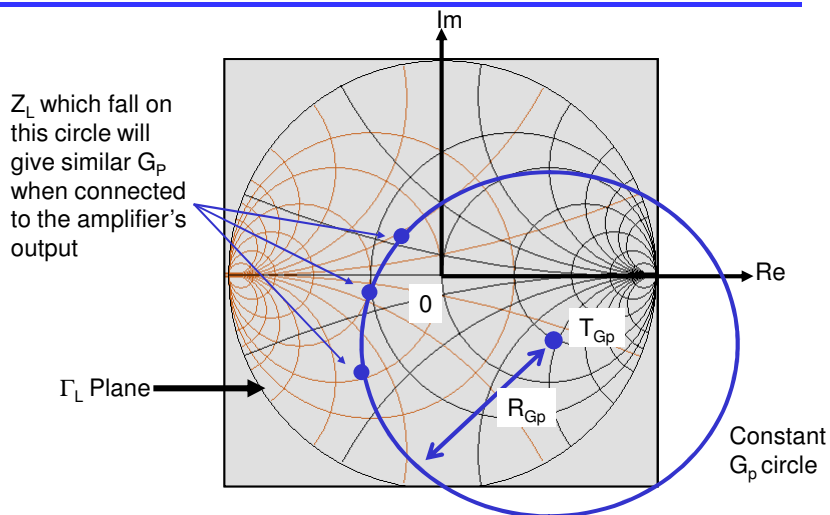
$$R_{GP} = \frac{\sqrt{1 - 2K|s_{12}s_{21}|g_2 + |s_{12}s_{21}|^2 g_2^2}}{|-1 - |s_{22}|^2 g_2 + |D|^2 g_2|} \quad (2.1d)$$

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## Constant Power Gain Circles (2)



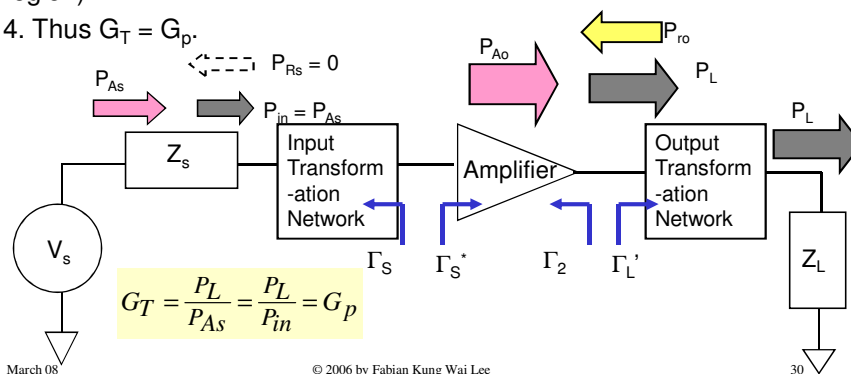
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## The Constant $G_p$ Circle Approach

- 1. Use constant  $G_p$  circles together with LSC (if amplifier is conditionally stable) to select the appropriate  $\Gamma_L$ .
- 2. From the selected  $\Gamma_L$  determine  $\Gamma_1$ .
- 3. Conjugate match the input port (make sure  $\Gamma_s$  is in the stable region).
- 4. Thus  $G_T = G_p$ .



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## Example 2.1 - Constant $G_p$ Amplifier Design

- A microwave BJT has  $S_{11}=0.9\angle 60^\circ$ ,  $S_{12}=0.06\angle 60^\circ$ ,  $S_{21}=3\angle 120^\circ$ ,  $S_{22}=0.82\angle -30^\circ$  at 1.2GHz. Design a single-stage amplifier having  $50\Omega$  input and output transmission lines, with a required transducer power gain between 40-50.
- Solution...
- $K = 0.902$  and  $|D|=0.898$ , so the amplifier is conditionally stable.
- S-parameters at 1.2GHz:

$$S_{11} = 0.45 + 0.7794i$$

$$S_{12} = 0.03 + 0.052i$$

$$S_{21} = -1.5 + 2.598i$$

$$S_{22} = 0.7101 - 0.41i$$

$$D = 0.8191 + 0.369i$$

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## Example 2.1 Cont...

Plot Load Stability Circle...

Since  $|S_{11}|=0.9 < 1$  and the LSC encloses the origin, the stable region is as indicated.

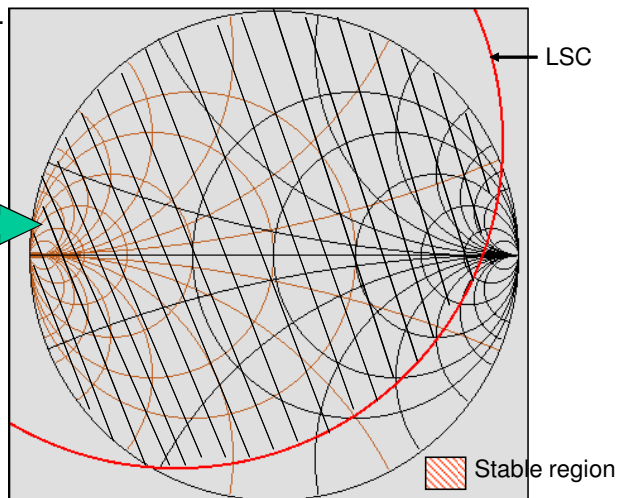
Plane for  $\Gamma_L$

$$CL := \frac{(S_{22} - D S_{11})}{(|S_{22}|)^2 - (|D|)^2}$$

$$CL = -0.4003 + 0.4631i$$

$$RL := \left| \frac{S_{12} S_{21}}{(|S_{22}|)^2 - (|D|)^2} \right|$$

$$RL = 1.336$$



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## Example 2.1 Cont...

Constant  $G_p$  Circle for  $G_p = 36$

$$C2 := S22 - D \cdot \overline{S11} \quad C2 = 0.0539 + 0.0624i$$

$$g2 := \frac{36}{(|S21|)^2} \quad g2 = 4$$

$$C_{gp} := \frac{1}{-1 - (|S22|^2 \cdot g2 + |D|^2 \cdot g2)} \cdot (-g2 \cdot \text{Re}(C2) + i \cdot g2 \cdot \text{Im}(C2))$$

$$C_{gp} = 0.4678 - 0.5413i$$

$$K := \frac{1 - (|S11|)^2 - (|S22|)^2 + (|D|)^2}{2 \cdot |S21 \cdot S12|}$$

$$K = 0.902 \quad -1 - (|S22|^2 \cdot g2 + |D|^2 \cdot g2) = -0.4611$$

$$R_{gp} := \frac{\sqrt{1 - 2 \cdot K \cdot |S12 \cdot S21| \cdot g2 + (|S12 \cdot S21 \cdot g2|)^2}}{|-1 - (|S22|^2 \cdot g2 + |D|^2 \cdot g2)|}$$

$$R_{gp} = 1.0161$$

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Constant  $G_p$  Circle for  $G_p = 45$

$$C2 := S22 - D \cdot \overline{S11} \quad C2 = 0.0539 + 0.0624i$$

$$g2 := \frac{45}{(|S21|)^2} \quad g2 = 5$$

$$C_{gp} := \frac{1}{-1 - (|S22|^2 \cdot g2 + |D|^2 \cdot g2)} \cdot (-g2 \cdot \text{Re}(C2) + i \cdot g2 \cdot \text{Im}(C2))$$

$$C_{gp} = 0.8262 - 0.9561i$$

$$K := \frac{1 - (|S11|)^2 - (|S22|)^2 + (|D|)^2}{2 \cdot |S21 \cdot S12|}$$

$$K = 0.902 \quad -1 - (|S22|^2 \cdot g2 + |D|^2 \cdot g2) = -0.3264$$

$$R_{gp} := \frac{\sqrt{1 - 2 \cdot K \cdot |S12 \cdot S21| \cdot g2 + (|S12 \cdot S21 \cdot g2|)^2}}{|-1 - (|S22|^2 \cdot g2 + |D|^2 \cdot g2)|}$$

$$R_{gp} = 1.3228$$

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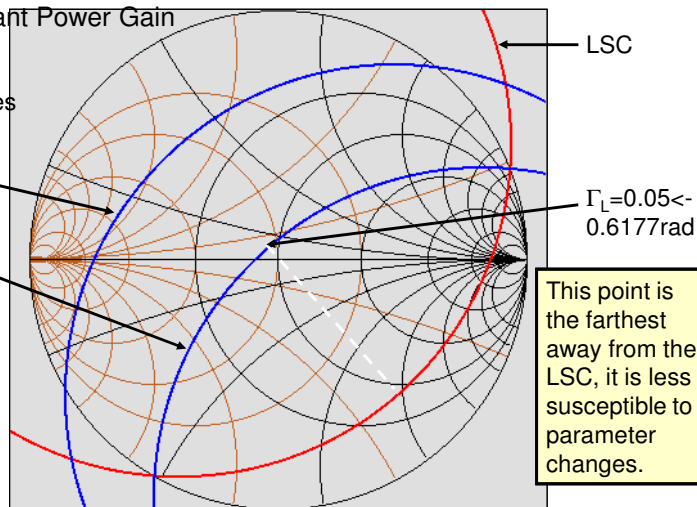
## Example 2.1 Cont...

Draw the Constant Power Gain Circles...

Constant  $G_p$  circles on the  $\Gamma_L$  plane

$G_p=36$

$G_p=45$



This point is the farthest away from the LSC, it is less susceptible to parameter changes.

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## Example 2.1 Cont...

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- Finding the corresponding load impedance

$$\Gamma_L = 0.05 \angle 0.6177 \text{ rad} = 0.041 - j0.029$$

$$Z_L = Z_o \frac{1 + \Gamma_L}{1 - \Gamma_L} \quad Z_o = 50$$

$$Z_L = 54.154 - j3.144$$

## Example 2.1 Cont...

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- Finding the corresponding input impedance and taking its complex conjugate:

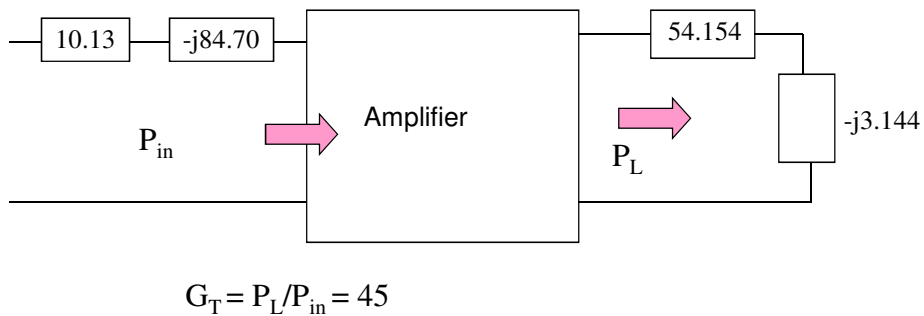
$$\Gamma_1 = \frac{S_{11} - D \Gamma_L}{1 - S_{22} \Gamma_L} = 0.4472 + j0.785$$

$$Z_1 = \frac{1 + \Gamma_1}{1 - \Gamma_1} Z_o = 10.129 + j84.704$$

$$Z_s = Z_1^* = 10.129 - j84.704$$

## Example 2.1 Cont...

- The block diagram...



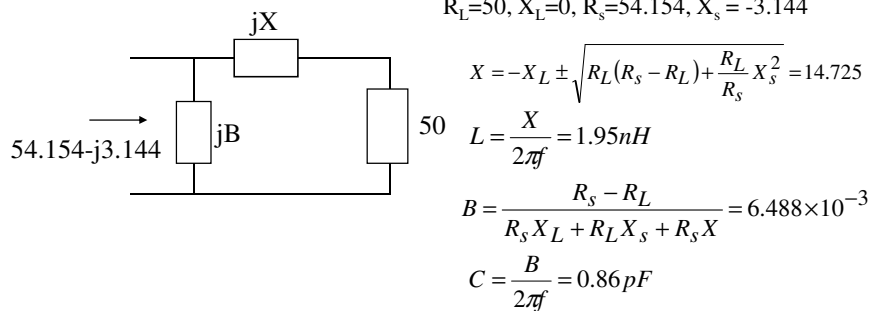
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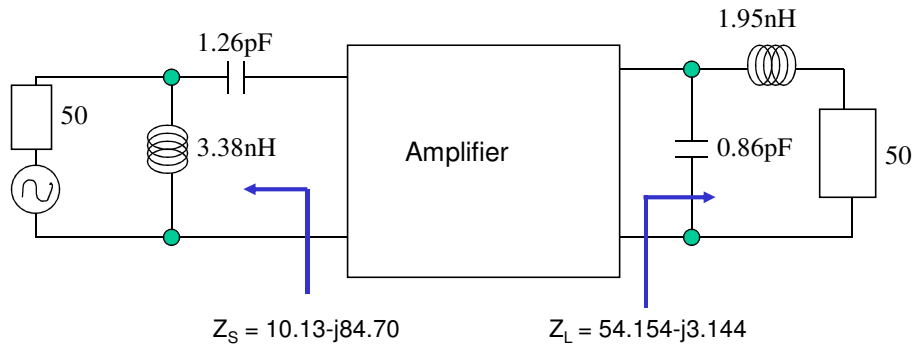
## Example 2.1 Cont...

- Since actual  $Z_s$  and  $Z_L$  are  $50\Omega$ , impedance transformation network is needed to change the  $50\Omega$  load impedance into  $54.154 - j3.144$ , and to  $10.13 - j84.70$  for source impedance. We may use either 2 or 3 elements impedance transformation network. This is shown below for the load network.



## Example 2.1 Cont...

- The final network...



$$G_T = P_L/P_{in} = 45$$

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## 3.0 Design for Fixed Transducer Power Gain $G_T$ – The Constant $G_A$ Circle Approach

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## Preliminary

- The approach is just the dual of the Constant  $G_P$  Approach.
- Instead of tuning  $\Gamma_L$  for fixed  $G_P$  we tune  $\Gamma_s$  for fixed  $G_A$ , therefore constant  $G_A$  circles and  $\Gamma_s$  plane for the Smith Chart are employed.
- This approach is useful we need to take into account the final transducer power gain  $G_T$  and noise figure of the amplifier.



## Constant Available Power Gain Circles (1)

- From the Dependency Diagram of previous chapter we see that  $G_A$  only depends on  $\Gamma_s$ . Indeed it is possible to determine the range of  $\Gamma_s$  which would give a fixed  $G_A$ , and this range actually falls on the locus of a circle on Smith Chart.
- The derivation is similar to the procedures in Appendix 1, see [4] for the details.

$$C_1 = s_{11} - Ds_{22}^* \quad (3.1a)$$

$$g_1 = \frac{G_A}{|s_{21}|^2} \quad (3.1b)$$

Normalized  $G_A$

Center of circle

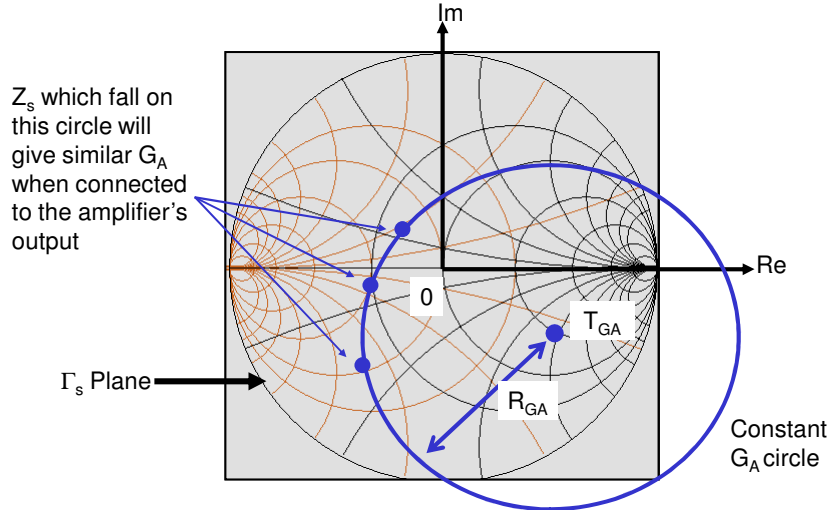
$$T_{G_A} = \frac{g_1 \operatorname{Re}(C_1)}{1 + |s_{11}|^2 g_1 - |D|^2 g_1} - j \frac{g_1 \operatorname{Im}(C_1)}{1 + |s_{11}|^2 g_1 - |D|^2 g_1} \quad (3.1c)$$

Radius of circle

$$R_{G_A} = \frac{\sqrt{1 - 2K|s_{12}s_{21}|g_1 + |s_{12}s_{21}|^2 g_1^2}}{|1 + |s_{11}|^2 g_1 - |D|^2 g_1|} \quad (3.1d)$$

## Constant Available Power Gain Circles (2)

Extra



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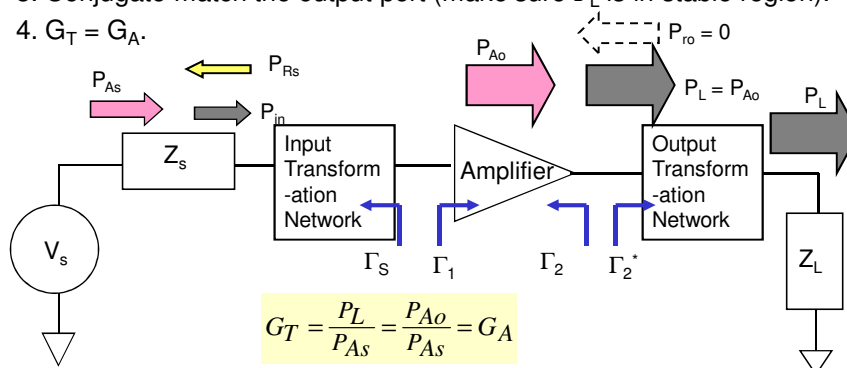
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## The Constant $G_A$ Circle Approach

Extra

1. Use constant  $G_A$  circles together with SSC (if amplifier is conditionally stable) to select the appropriate  $\Gamma_s$ .
2. From the selected  $\Gamma_s$  determine  $\Gamma_2$ .
3. Conjugate match the output port (make sure  $\Gamma_L$  is in stable region).
4.  $G_T = G_A$ .



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