7- Small-Signal Amplifier

Theory

References

- [1]* D.M. Pozar, “Microwave engineering”, 3rd Edition, 2005 John-Wiley & Sons.
- [2] R.E. Collin, “Foundations for microwave engineering”, 2nd Edition, 1992 McGraw-Hill.
- [3] R. Ludwig, P. Bretchko, “RF circuit design - theory and applications”, 2000 Prentice-Hall.
- [4]* G. Gonzalez, “Microwave transistor amplifiers - analysis and design”, 2nd Edition 1997, Prentice-Hall.
- [5] G. D. Vendelin, A. M. Pavio, U. L. Rhode, “Microwave circuit design - using linear and nonlinear techniques”, 1990 John-Wiley & Sons. A more updated version of this book, published in May 1992 is also available.
- [6]* Gilmore R., Besser L.,”Practical RF circuit design for modern wireless systems”, Vol. 1 & 2, 2003, Artech House.

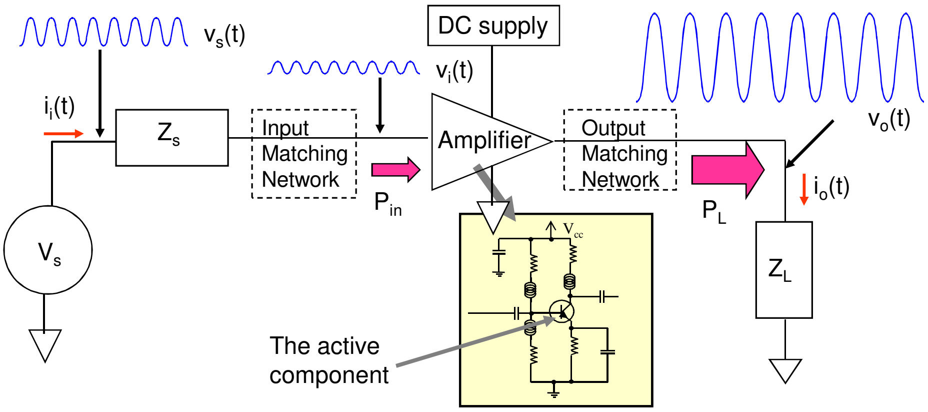
*Recommended

Agenda

- Basic amplifier concepts and small-signal amplifier.
- Amplifier characteristics.
- Basic amplifier block diagram and power gain expressions.
- Dependency of gain on amplifier parameters.
- Stability concepts and criteria.
- Stability circles and regions.
- Stability Factor.
- Stabilization methods.

1.0 Basic Amplifier Concepts

General Amplifier Block Diagram



Input and output voltage relation of the amplifier can be modeled simply as:

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) + H.O.T.$$

Amplifier Classification

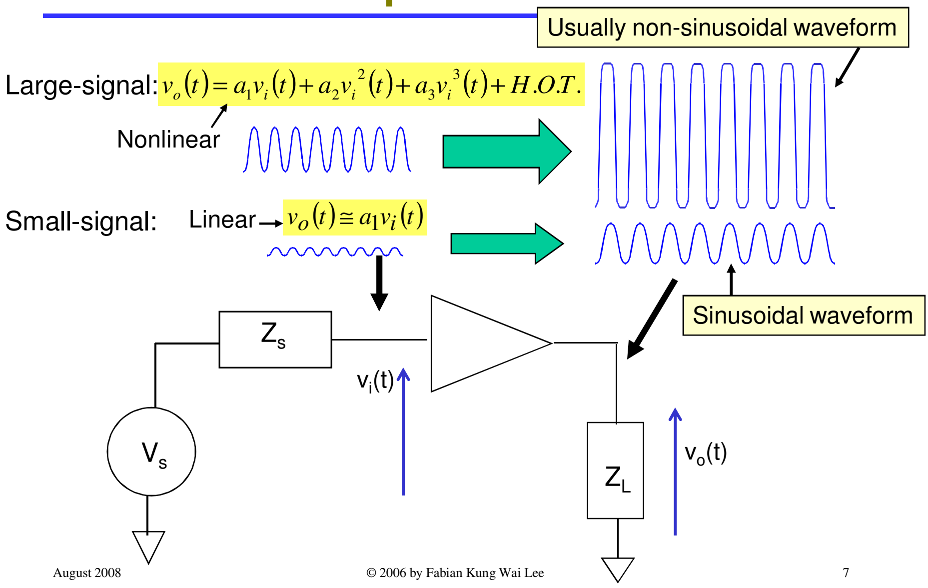
- Amplifier can be categorized in 2 manners.
- According to signal level:
  - Small-signal Amplifier.
  - Power/Large-signal Amplifier.

Our approach in this chapter

- According to D.C. biasing scheme of the active component:
  - Class A.
  - Class B.
  - Class AB.
  - Class C.

There are also other classes, such as Class D (D stands for digital), Class E and Class F. These all uses the transistor/FET as a switch.

Small-Signal Versus Large-Signal Operation



Usually non-sinusoidal waveform

Large-signal: $v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) + H.O.T.$

Nonlinear

Small-signal: Linear $\rightarrow$ $v_o(t) \cong a_1 v_i(t)$

Sinusoidal waveform

Small-Signal Amplifier (SSA)

- All amplifiers are inherently nonlinear.
- However when the input signal is small, the input and output relationship of the amplifier is approximately linear.

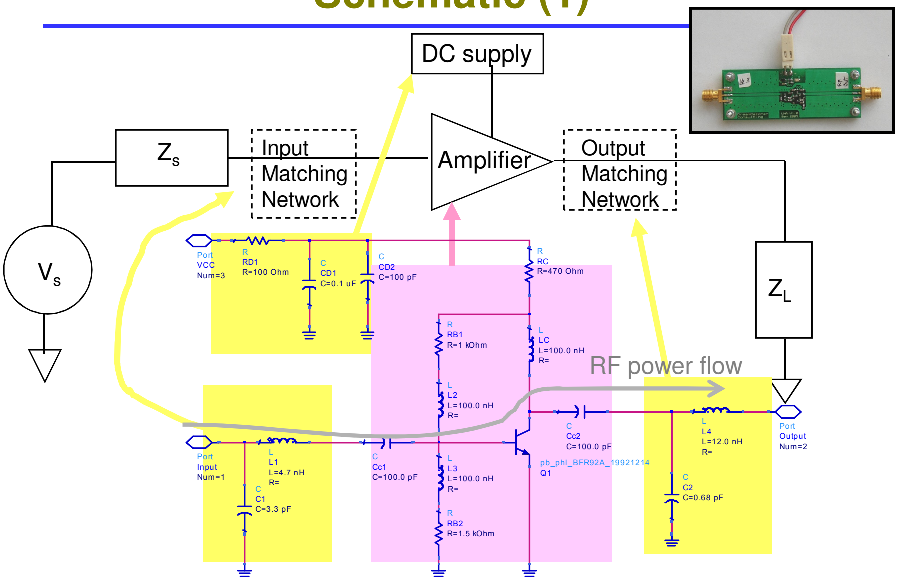
$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) + H.O.T. \cong a_1 v_i(t)$$

Linear relation

When $v_i(t) \rightarrow 0$ (< 2.6mV) $\longrightarrow$ $v_o(t) \cong a_1 v_i(t)$ ***(1.1)***

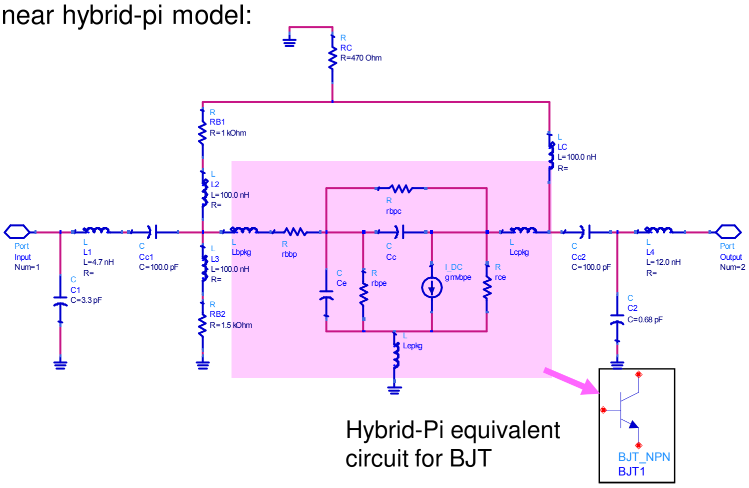
- This linear relationship applies also to current and power.
- An amplifier that fulfills these conditions: (1) small-signal operation (2) linear, is called Small-Signal Amplifier (SSA). SSA will be our focus.
- If a SSA amplifier contains BJT and FET, these components can be replaced by their respective small-signal model, for instance the hybrid-Pi model for BJT.

Example 1.1 - An RF Amplifier Schematic (1)



Example 1.1 Cont…

- Under AC and small-signal conditions, the BJT can be replaced with linear hybrid-pi model:



Hybrid-Pi equivalent circuit for BJT

Typical RF Amplifier Characteristics

- To determine the performance of an amplifier, the following characteristics are typically observed.
- 1. Power Gain.
- 2. Bandwidth (operating frequency range).
- 3. Noise Figure.
- 4. Phase response.
- 5. Gain compression.
- 6. Dynamic range.
- 7. Harmonic distortion.
- 8. Intermodulation distortion.
- 9. Third order intercept point (TOI).

Important to small-signal amplifier

Important parameters of large-signal amplifier (Related to Linearity)

Will elaborate in “High-Power Circuits”

Power Gain

- For amplifiers functioning at RF and microwave frequencies, usually of interest is the input and output power relation.
- The ratio of output power over input power is called the **Power Gain (G)**, usually expressed in dB.

Power Gain $$G = 10\log_{10}\left(\frac{\text{Output Power}}{\text{Input Power}}\right)\ \text{dB} \quad \textit{(1.2)}$$

- There are a number of definitions for power gain as we will see shortly.
- Furthermore G is a function of frequency and the input signal level.

Why Power Gain for RF and Microwave Circuits? (1)

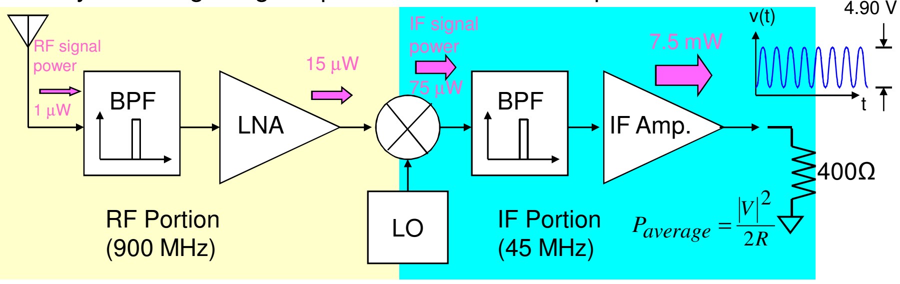
- Power gain is preferred for high frequency amplifiers as the impedance encountered is usually low (due to presence of parasitic capacitance).

Power = Voltage x Current

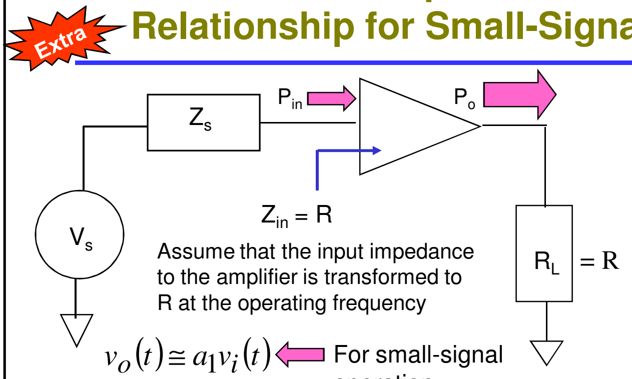
- For instance if the amplifier is required to drive 50Ω load the voltage across the load may be small, although the corresponding current may be large (there is current gain).
- For amplifiers functioning at lower frequency (such as IF frequency), it is the voltage gain that is of interest, since impedance encountered is usually higher (less parasitic).
- For instance if the output of an IF amplifier drives the demodulator circuits, which are usually digital systems, the impedance looking into the digital system is high and large voltage can developed across it. Thus working with voltage gain is more convenient.

Why Power Gain for RF and Microwave Circuits? (2)

- Instead of focusing on voltage or current gain, RF engineers focus on power gain.
- By working with power gain, the RF designer is free from the constraint of system impedance. For instance in the simple receiver block diagram below, each block contribute some power gain. A large voltage signal can be obtained from the output of the final block by attaching a high impedance load to it's output.



Derivation of Input and Output Power Relationship for Small-Signal Operation



$Z_{in} = R$

Assume that the input impedance to the amplifier is transformed to R at the operating frequency

- Usually we express power in logarithmic scale, i.e. the dB or dBm scale.
- Here the relation between input and output power is in dB.

$$v_o(t) \cong a_1 v_i(t)$$

For small-signal operation

$$\frac{1}{2R}{v_o}^2 \cong {a_1}^2 \frac{1}{2R}{v_i}^2$$

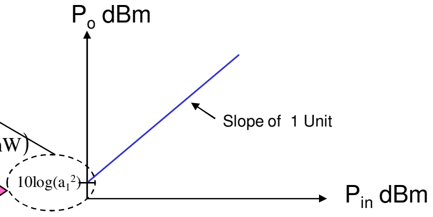
$$\Rightarrow P_o \cong a_1^2 P_{in}$$

$$\Rightarrow P_o/1\text{mW} \cong a_1^2(P_{in}/1\text{mW})$$

$$\Rightarrow 10\cdot\log(P_o/1\text{mW}) \cong 10\cdot\log\left(a_1^2\right) + 10\cdot\log(P_{in}/1\text{mW})$$

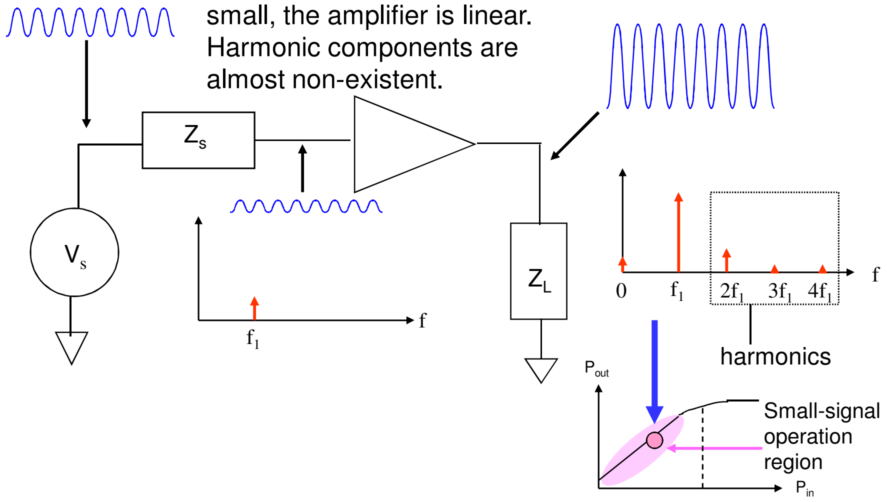
$$\Rightarrow \boxed{P_o dBm \cong 10\cdot\log\left(a_1^2\right) + P_{in} dBm}$$

Power gain



Harmonic Distortion (1)

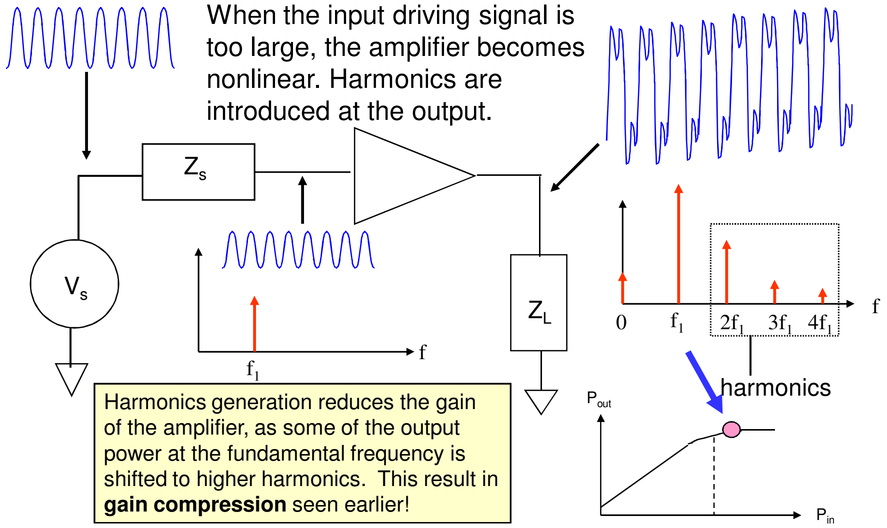
When the input driving signal is small, the amplifier is linear. Harmonic components are almost non-existent.



Small-signal operation region

Harmonic Distortion (2)

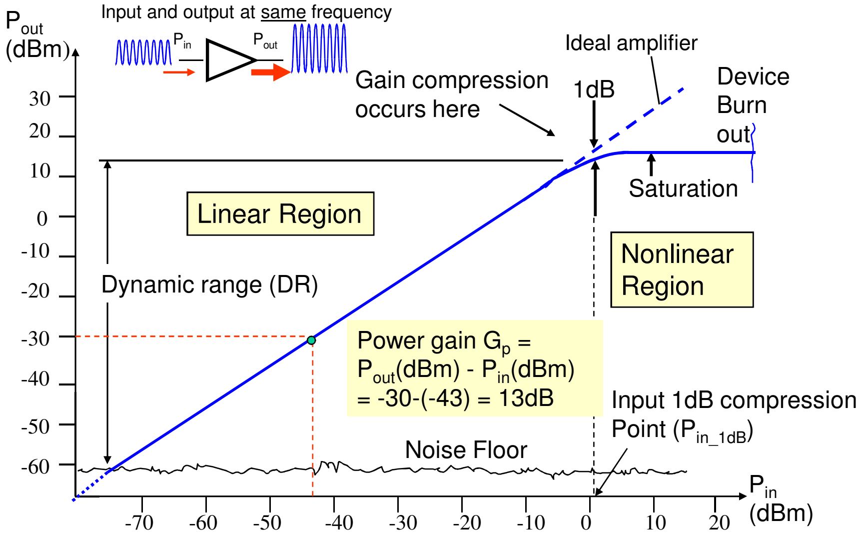
When the input driving signal is too large, the amplifier becomes nonlinear. Harmonics are introduced at the output.



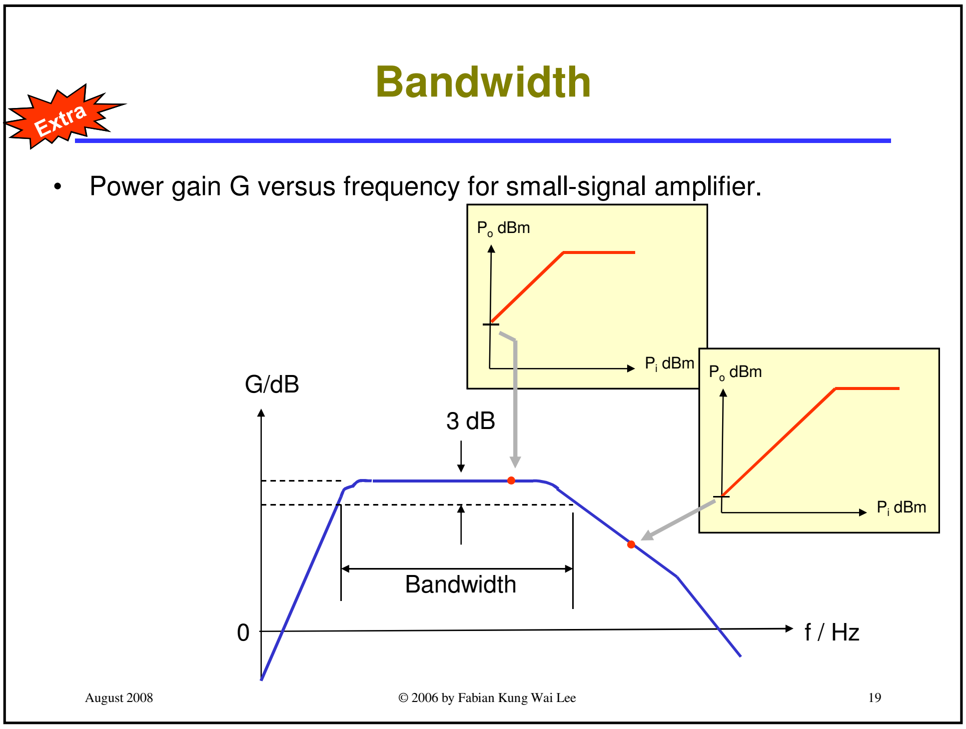
Harmonics generation reduces the gain of the amplifier, as some of the output power at the fundamental frequency is shifted to higher harmonics. This result in **gain compression** seen earlier!

Power Gain, Dynamic Range and Gain Compression

Input and output at same frequency

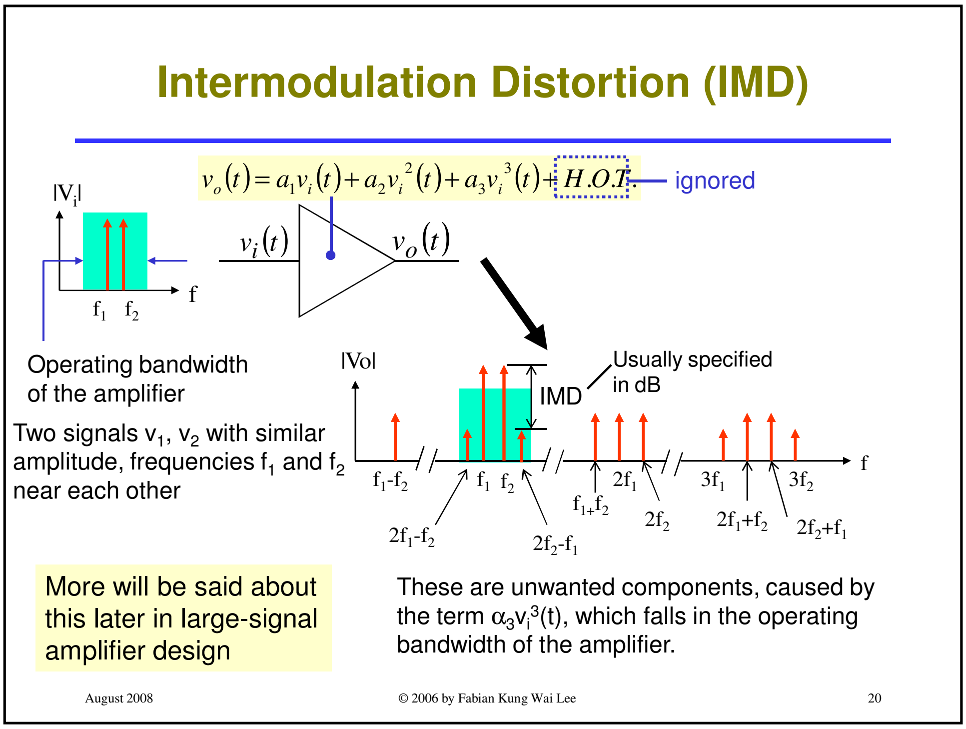


Power gain G_p = P_{out}(dBm) - P_{in}(dBm) = -30-(-43) = 13dB



Bandwidth

- Power gain G versus frequency for small-signal amplifier.



Intermodulation Distortion (IMD)

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) + H.O.T.$$

Operating bandwidth of the amplifier

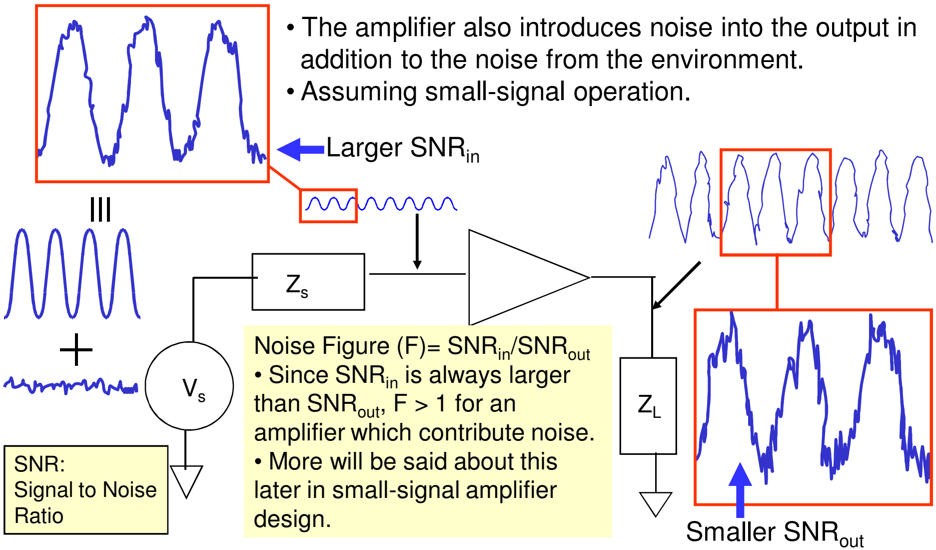
Two signals v_1, v_2 with similar amplitude, frequencies f_1 and f_2 near each other

More will be said about this later in large-signal amplifier design

These are unwanted components, caused by the term $\alpha_3 v_i^3(t)$, which falls in the operating bandwidth of the amplifier.

Noise Figure (F)

- The amplifier also introduces noise into the output in addition to the noise from the environment.
- Assuming small-signal operation.



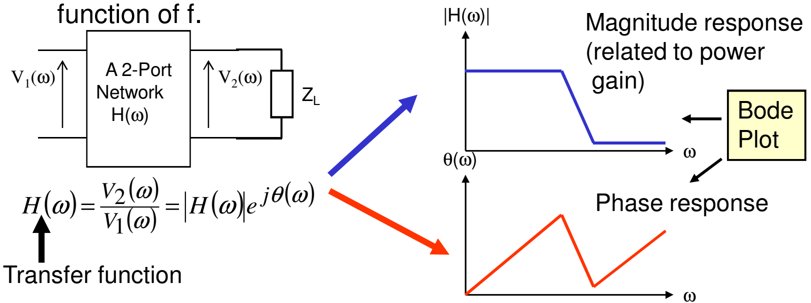
Noise Figure (F)= SNR$_{in}$/SNR$_{out}$
- Since SNR$_{in}$ is always larger than SNR$_{out}$, F > 1 for an amplifier which contribute noise.
- More will be said about this later in small-signal amplifier design.

SNR:
Signal to Noise Ratio

Phase Response (1)

Extra

- Phase consideration is important for amplifier working with wideband signals.
- For a signal to be amplified with no distortion, 2 requirements are needed (from linear systems theory).
  - 1. The magnitude of the power gain transfer function must be a constant with respect to frequency f.
  - 2. The phase of the power gain transfer function must be a linear function of f.



$$H(\omega)=\frac{V_2(\omega)}{V_1(\omega)}=|H(\omega)|e^{j\theta(\omega)}$$

Transfer function

Magnitude response (related to power gain)

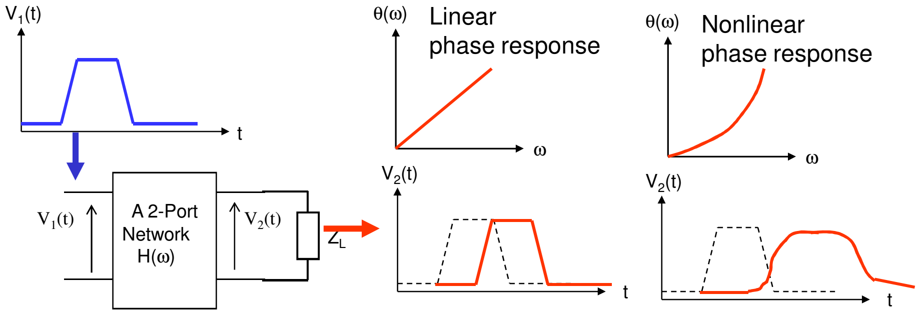
Bode Plot

Phase response

Extra

Phase Response (2)

- A linear phase produces a constant time delay for all signal frequencies, and a nonlinear phase shift produces different time delay for different frequencies.
- Property (1) means that all frequency components will be amplified by similar amount, property (2) implies that all frequency components will be delayed by similar amount.



2.0 Small-Signal Amplifier Power Gain Expressions

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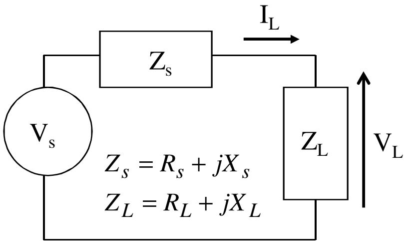
The Theory of Maximum Power Transfer (1)



$$Z_s = R_s + jX_s$$
$$Z_L = R_L + jX_L$$

Basic source-load network

Time averaged power dissipated across load Z_L: $P_L = \frac{1}{2}\mathrm{Re}\left\{V_L I_L^{\ *}\right\}$

where $V_L = \frac{V_s Z_L}{Z_s + Z_L}$ $\quad I_L = \frac{V_s}{Z_s + Z_L}$

$$P_L = \frac{1}{2}\mathrm{Re}\left\{\frac{V_s Z_L}{Z_s + Z_L} \cdot \left(\frac{V_s}{Z_s + Z_L}\right)^*\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{|V_s|^2 Z_L}{|Z_s + Z_L|^2}\right\}$$

$$\Rightarrow P_L = \frac{1}{2}\frac{|V_s|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

Letting $\frac{\partial P_L}{\partial R_L} = \frac{\partial P_L}{\partial X_L} = 0$

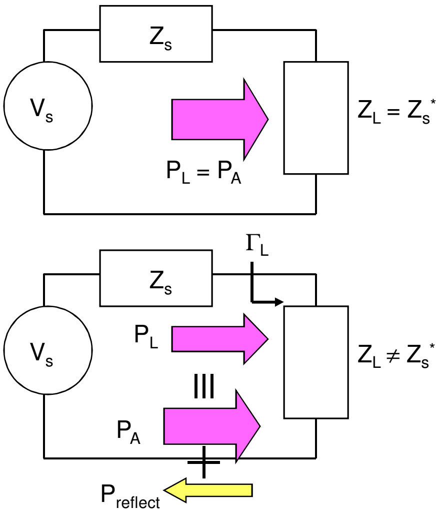
$$P_L = P_L(R_L, X_L)$$

We find that the value for R_L and X_L that would maximize P_L is $R_L = R_s$, $X_L = -X_s$.
In other words: $Z_L = Z_s^*$

To maximize power transfer to the load impedance, Z_L must be the complex conjugate of Z_s, a notion known as Conjugate Matched.

Extra

The Theory of Maximum Power Transfer (2)



Under conjugate match condition:

$$P_L(\max) = \frac{|V_s|^2}{8R_s} = P_A$$ ← Available Power

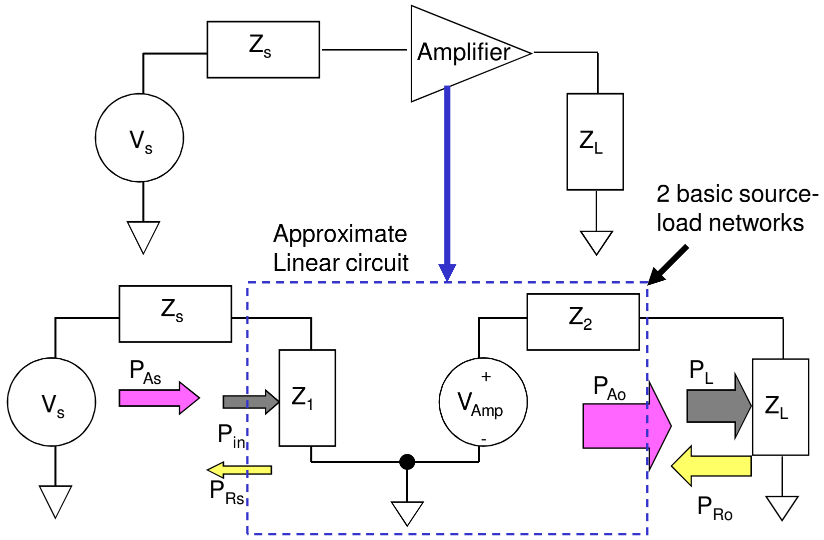
Under non-conjugate match condition:

$$P_L < \frac{|V_s|^2}{8R_s} = P_A - P_{\mathrm{Reflect}}$$

or $P_L = P_A\left(1 - |\Gamma_L|^2\right)$

We can consider the load power P_L to consist of the available power P_A minus the reflected power $P_{reflect}$.

Power Components in an Amplifier



Power Gain Definition

- From the power components, 3 types of power gain can be defined.

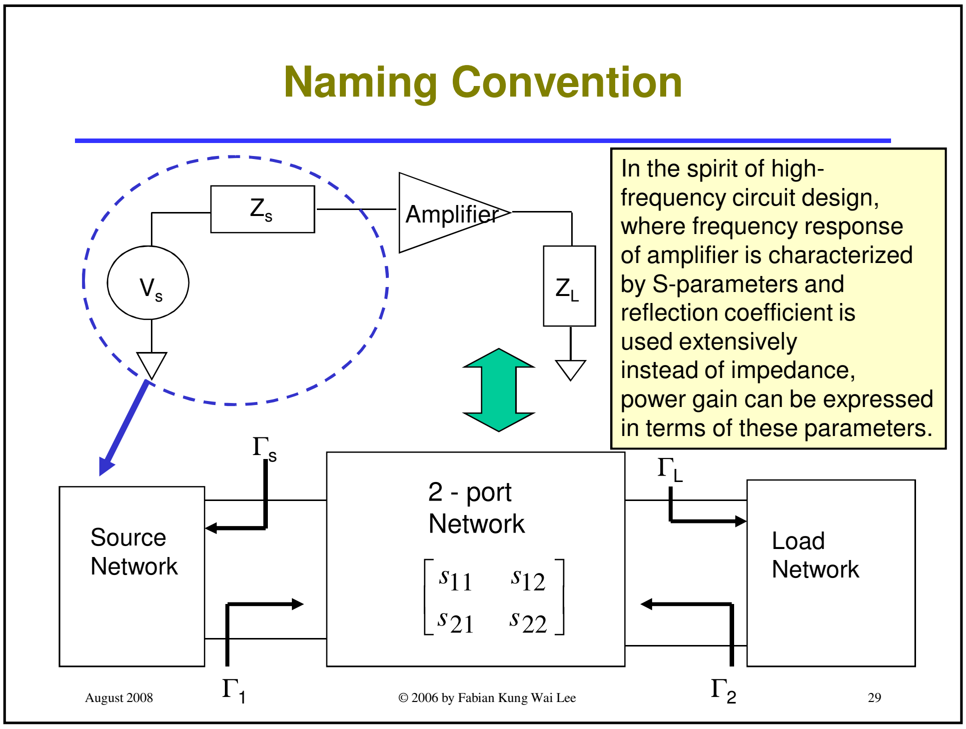
$$\text{Power Gain } G_p = \frac{\text{Power delivered to load}}{\text{Input power to Amp.}} = \frac{P_L}{P_{in}} \quad \textit{(2.1a)}$$

$$\text{Available Power Gain } G_A = \frac{\text{Available load Power}}{\text{Available Input power}} = \frac{P_{Ao}}{P_{As}} \quad \textit{(2.1b)}$$

$$\text{Transducer Gain } \underset{\uparrow}{G_T} = \frac{\text{Power delivered to load}}{\text{Available Input power}} = \frac{P_L}{P_{As}} \quad \textit{(2.1c)}$$

The effective power gain

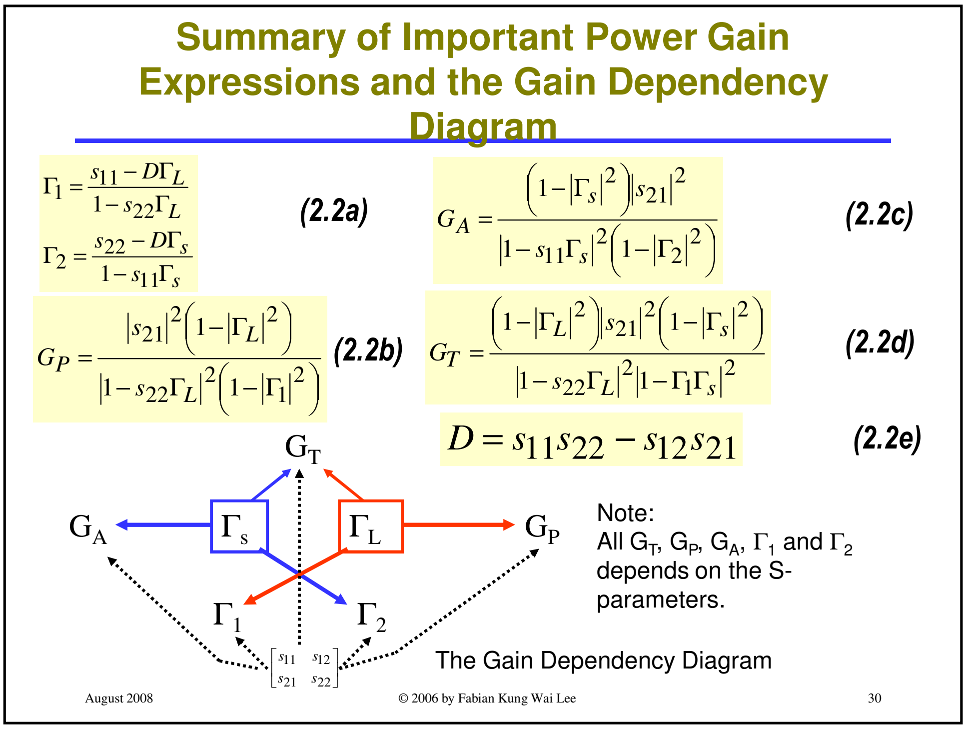
- G_P, G_A and G_T can be expressed as the S-parameters of the amplifier and the reflection coefficients of the source and load networks. Refer to Appendix 1 for the derivation.



Naming Convention

$$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

In the spirit of high-frequency circuit design, where frequency response of amplifier is characterized by S-parameters and reflection coefficient is used extensively instead of impedance, power gain can be expressed in terms of these parameters.



Summary of Important Power Gain Expressions and the Gain Dependency Diagram

$$\Gamma_1 = \frac{s_{11} - D\Gamma_L}{1 - s_{22}\Gamma_L} \qquad \Gamma_2 = \frac{s_{22} - D\Gamma_s}{1 - s_{11}\Gamma_s} \quad \textbf{(2.2a)}$$

$$G_P = \frac{|s_{21}|^2\left(1 - |\Gamma_L|^2\right)}{|1 - s_{22}\Gamma_L|^2\left(1 - |\Gamma_1|^2\right)} \quad \textbf{(2.2b)}$$

$$G_A = \frac{\left(1 - |\Gamma_s|^2\right)|s_{21}|^2}{|1 - s_{11}\Gamma_s|^2\left(1 - |\Gamma_2|^2\right)} \quad \textbf{(2.2c)}$$

$$G_T = \frac{\left(1 - |\Gamma_L|^2\right)|s_{21}|^2\left(1 - |\Gamma_s|^2\right)}{|1 - s_{22}\Gamma_L|^2|1 - \Gamma_1\Gamma_s|^2} \quad \textbf{(2.2d)}$$

$$D = s_{11}s_{22} - s_{12}s_{21} \quad \textbf{(2.2e)}$$

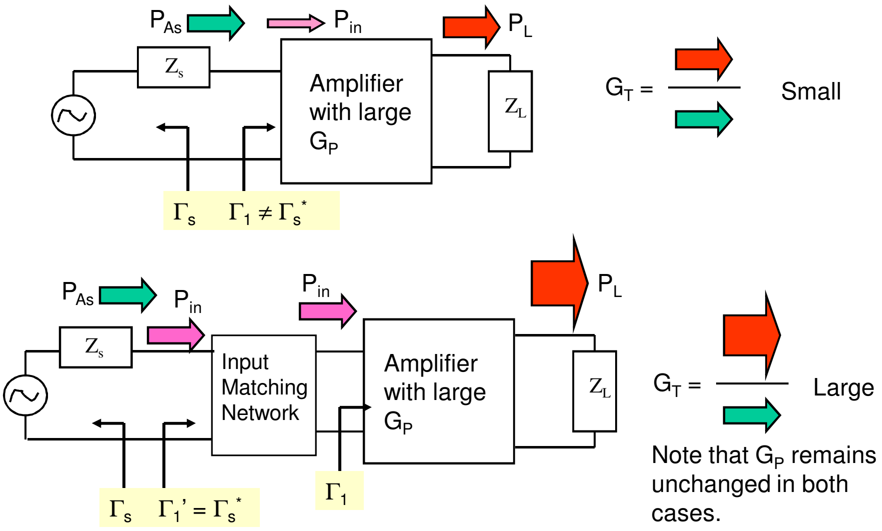
Note: All G_T, G_P, G_A, Γ_1 and Γ_2 depends on the S-parameters.

The Gain Dependency Diagram

Transducer Power Gain G_T (1)

- G_T is the relevant indicator of the amplifying capability of the amplifier.
- Whenever we design an amplifier to a specific power gain, we refer to the transducer power gain G_T.
- G_P and G_A are usually used in the process of amplifier synthesis to meet a certain G_T.
- An amplifier can have a large G_P or G_A and yet small G_T, as illustrated in the next slide.

Transducer Power Gain G_T (2)



Note that G_P remains unchanged in both cases.

Essence of Small-Signal Amplifier Design

- **In essence, designing a small-signal amplifier with transistor or monolithic microwave integrated circuit (MMIC) implies finding the suitable load and source impedance to be connected to the output and input port, so that you get the required transducer power gain G_T, bandwidth and other characteristics.**

Extra

Unilateral Condition (1)

- In certain cases, when operating frequency is low, $|s_{12}| \to 0$. Such condition is known as Unilateral.
- Under unilateral condition: $\Gamma_1 = \frac{s_{11} - D\Gamma_L}{1 - s_{22}\Gamma_L} = \frac{s_{11} - s_{11}s_{22}\Gamma_L}{1 - s_{22}\Gamma_L} = s_{11}$
- From (2.2d):

$$G_T = \underbrace{\boxed{\frac{1-|\Gamma_s|^2}{|1-s_{11}\Gamma_s|^2}}}_{G_s} \cdot \underbrace{|s_{21}|^2}_{G_o} \cdot \underbrace{\boxed{\frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2}}}_{G_L} = G_s G_o G_L \qquad \textbf{(2.3)}$$

- We see that the Transducer Power Gain G_T consists of 3 parts that are independent of each other.

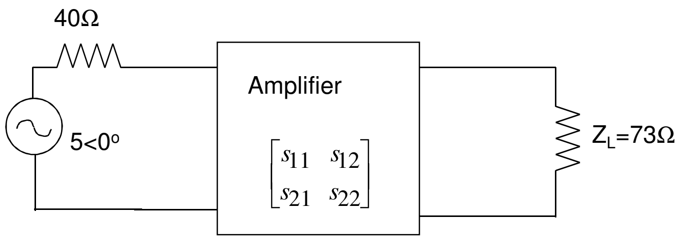
Extra

Unilateral Condition (2)

- In small-signal amplifier design for unilateral condition, we can find suitable source and load impedance for a required G_T by optimizing G_s and G_L independently, and this simplifies the design procedures.
- However in most case, s_{12} is typically not zero especially at frequency above 1 GHz. Thus we will not pursue design techniques for unilateral condition.

Example 2.1 – Familiarization with the Gain Expressions

- An RF amplifier has the following S-parameters at f_o: $s_{11}=0.3\angle -70^o$, $s_{21}=3.5\angle 85^o$, $s_{12}=0.2\angle -10^o$, $s_{22}=0.4\angle -45^o$. The system is shown below. Assuming reference impedance (used for measuring the S-parameters) $Z_o=50\Omega$, find:
- (a) G_T, G_A, G_P.
- (b) P_L, P_A, P_{inc}.



Example 2.1 Cont...

- Step 1 - Find Γ_s and Γ_L .
- Step 2 - Find Γ_1 and Γ_2 .
- Step 3 - Find G_T, G_A, G_P.
- Step 4 - Find P_L, P_A.

$$\Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o} = -0.111 \qquad \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = 0.187$$

$$\Gamma_1 = \frac{s_{11} - D\Gamma_L}{1 - s_{22}\Gamma_L} = 0.146 - j0.151$$

$$\Gamma_2 = \frac{s_{22} - D\Gamma_s}{1 - s_{11}\Gamma_s} = 0.265 - j0.358$$

$$G_P = \frac{|s_{21}|^2\left(1 - |\Gamma_L|^2\right)}{|1 - s_{22}\Gamma_L|^2\left(1 - |\Gamma_1|^2\right)} = 13.742$$

$$G_A = \frac{\left(1 - |\Gamma_s|^2\right)|s_{21}|^2}{|1 - s_{11}\Gamma_s|^2\left(1 - |\Gamma_2|^2\right)} = 14.739$$

$$G_T = \frac{\left(1 - |\Gamma_L|^2\right)|s_{21}|^2\left(1 - |\Gamma_s|^2\right)}{|1 - s_{22}\Gamma_L|^2|1 - \Gamma_1\Gamma_s|^2} = 12.562$$

$$P_A = \frac{|V_s|^2}{8 \cdot \mathrm{Re}[Z_s]} = 0.078W$$

Try to derive
These 2 relations

$$P_{in} = P_A\left(1 - \left|\frac{Z_1 - Z_s}{Z_1 + Z_s}\right|^2\right) = 0.0714W$$

$$P_L = G_P P_{in} = 0.9814W$$

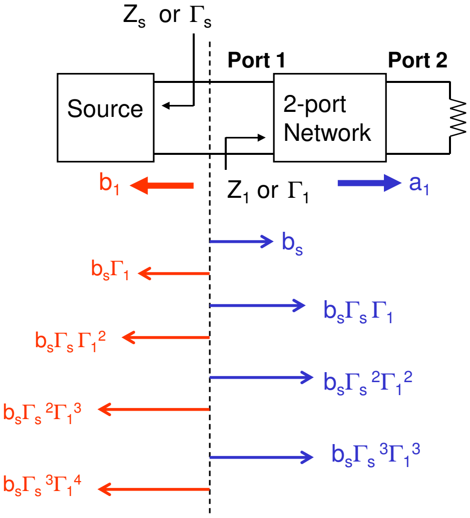
Again note that this is an analysis problem.

3.0 Stability Analysis of Small-Signal Amplifier

Introduction

- An amplifier is a circuit designed to enlarged electrical signals.
- When there is no input, there should be no output, this condition is known as stable.
- On the contrary, if the amplifier produces an output when there is no input, it is unstable. In fact the amplifier becomes an oscillator!
- Thus a stability analysis is required to determine whether an amplifier circuit is stable or not.
- Stability analysis is also carried out by assuming a small-signal amplifier, since the initial signal that causes oscillation is always very small.
- Stability of an amplifier is affected by the load and source impedance connected to its two ports.
- An unstable or marginally stable amplifier can be made more stable.

Stability Concept (1) - Perspective of Oscillation from Wave Propagation



- An incident wave V^+ propagating towards port 1 will suffer multiple reflection.
- If $|\Gamma_s\Gamma_1|>1$, then the magnitude of the incident wave towards port 1 will increase indefinitely, leading to oscillation.

A geometric series

$$\boxed{\begin{aligned} a_1 &= b_s + b_s\Gamma_1\Gamma_s + b_s\Gamma_1^2\Gamma_s^2 + \ldots \\ \Rightarrow a_1 &= \frac{b_s}{1-\Gamma_1\Gamma_s} \end{aligned}}$$

Only if $|\Gamma_1\Gamma_s|<1$

$$\boxed{\begin{aligned} b_1 &= b_s\Gamma_1 + b_s\Gamma_1^2\Gamma_s + b_s\Gamma_1^3\Gamma_s^2 + \ldots \\ \Rightarrow b_1 &= \frac{b_s\Gamma_1}{1-\Gamma_1\Gamma_s} \end{aligned}}$$

Stability Concept (2)

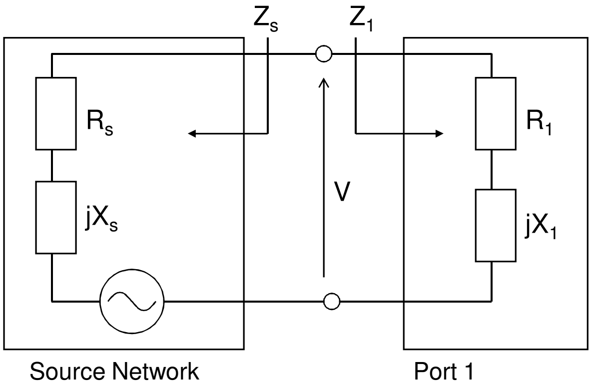
- Thus oscillation will occur when $|\Gamma_s \Gamma_1| > 1$.
- Since the source network is usually passive, $|\Gamma_s| < 1$. Thus for instability to occur, $|\Gamma_1| > 1$, this condition represents the potential for oscillation.
- Similar argument can be applied to port 2, and we see that the condition for oscillation is $|\Gamma_2| > 1$.
- Since input and output power of a 2-port network are related, when either port is stable, the other will also be stable.

Port 1:
$$|\Gamma_s \Gamma_1| = |\Gamma_s||\Gamma_1| > 1$$
$$\Rightarrow |\Gamma_1| > 1 \;\; \text{since} \;\; |\Gamma_s| \;\; \text{always} < 1$$

Port 2:
$$|\Gamma_L \Gamma_2| = |\Gamma_L||\Gamma_2| > 1$$
$$\Rightarrow |\Gamma_2| > 1 \;\; \text{since} \;\; |\Gamma_L| \;\; \text{always} < 1$$

Perspective of Oscillation from Circuit Theory (1)

Extra



$$V = \frac{R_1 + jX_1}{R_1 + R_s + j(X_1 + X_s)} \cdot V_s = \frac{Z_1}{Z_s + Z_1} V_s \qquad \textit{(3.1)}$$

Perspective of Oscillation from Circuit Theory (2)

Extra

Using Laplace transformation:

$$V(s)=\frac{Z_1(s)}{Z_s(s)+Z_1(s)}V_s(s)$$

For a system to oscillate, the denominator of the equation must has a conjugate poles at the frequency ω_o where oscillation occurs, this means:

$$Z_s(s)+Z_1(s)|_{s=j\omega_o}=0 \quad \textbf{\textit{(3.2a)}}$$

$$R_s+R_1|_{\omega_o}=0 \quad \textbf{\textit{(3.2b)}}$$

$$X_s+X_1|_{\omega_o}=0 \quad \textbf{\textit{(3.2c)}}$$

This fact will be used in later chapter to create oscillating circuit.

As will be shown, this requirement is similar to $|\Gamma_s\Gamma_1|=1$

Similarity Between Both Perspectives (1)

Extra

From: $$\Gamma=\frac{(R-Z_o)+jX}{(R+Z_o)+jX}$$

$$\Gamma_1\Gamma_s\big|_{\omega_o}=\frac{(R_1-Z_o)+jX_1}{(R_1+Z_o)+jX_1}\cdot\frac{(R_s-Z_o)+jX_s}{(R_s+Z_o)+jX_s}$$

$$=\frac{R_1R_s-Z_o(R_1+R_s)+Z_o^2-X_1X_s+j(R_1X_s+R_sX_1-Z_o(X_1+X_s))}{R_1R_s+Z_o(R_1+R_s)+Z_o^2-X_1X_s+j(R_1X_s+R_sX_1+Z_o(X_1+X_s))}$$

$$\left|\Gamma_1\Gamma_s\right|_{\omega_o}=\frac{\sqrt{\left(R_1R_s-Z_o(R_1+R_s)+Z_o^2-X_1X_s\right)^2+\left(R_1X_s+R_sX_1-Z_o(X_1+X_s)\right)^2}}{\sqrt{\left(R_1R_s+Z_o(R_1+R_s)+Z_o^2-X_1X_s\right)^2+\left(R_1X_s+R_sX_1+Z_o(X_1+X_s)\right)^2}}$$

Similarity Between Both Perspectives (2)

Extra

- We see that when $R_1+R_s|_{\omega o}= 0$, $X_1+X_s|_{\omega o}=0$:
  - $|\Gamma_1\Gamma_s|_{\omega o} = 1$
- And when $R_1+R_s|_{\omega o}<0$, $X_1+X_s|_{\omega o}= 0$ we have $|\Gamma_1\Gamma_s|_{\omega o} > 1$

- The above requirement also applies to port 2, in which case when $R_2+R_L|_{\omega o}=0$, $X_2+X_L|_{\omega o}=0$
  - $|\Gamma_2\Gamma_L|_{\omega o} = 1$
- And when $R_2+R_L|_{\omega o}<0$, $X_2+X_L|_{\omega o}= 0$ we have $|\Gamma_2\Gamma_L|_{\omega o} > 1$

Important Summary on Oscillation

- Assuming that $|\Gamma_s|$ and $|\Gamma_L|$ always < 1 for passive components, we conclude that:
- For a 2-port network to be stable, it is necessary that the load and source impedance are chosen in such a way that $|\Gamma_1| < 1$and $|\Gamma_2| < 1$.

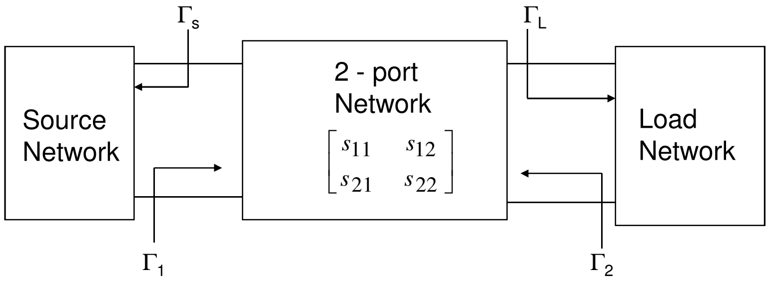
To prevent oscillation:

$$\left|\Gamma_1(\omega)\right| < 1$$

$$\left|\Gamma_2(\underset{\uparrow}{\omega})\right| < 1$$

The range of frequency of interest

Stability Criteria for Amplifier



- As seen previously, for stability $|\Gamma_1| < 1$ and $|\Gamma_2| < 1$.
- Using (2.2a) this can be expressed as:

$$\left|\Gamma_1\right| = \left|s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22}\Gamma_L}\right| < 1 \qquad \textbf{\textit{(3.3a)}}$$

$$\left|\Gamma_2\right| = \left|s_{22} + \frac{s_{12}s_{21}\Gamma_s}{1 - s_{11}\Gamma_s}\right| < 1 \qquad \textbf{\textit{(3.3b)}}$$

3.1 Stability Circles

Load Stability Circle (LSC)

- Setting $|\Gamma_1|=1$, we can determine all the corresponding values of Γ_L. The Γ_L happens to fall on the locus of a circle on Smith Chart.

$$\left|s_{11}+\frac{s_{12}s_{21}\Gamma_L}{1-s_{22}\Gamma_L}\right|=1$$

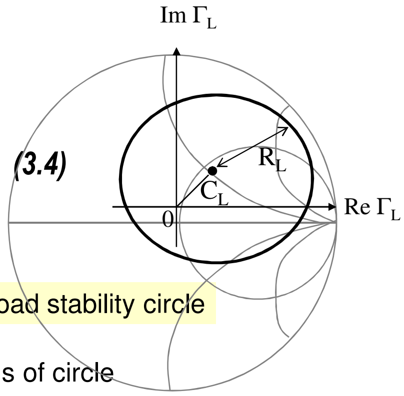
$$\Rightarrow\left|\Gamma_L-\frac{\left(s_{22}-Ds_{11}^{\ *}\right)^*}{|s_{22}|^2-|D|^2}\right|=\left|\frac{s_{12}s_{21}}{|s_{22}|^2-|D|^2}\right| \qquad \textbf{(3.4)}$$

$$\Rightarrow|\Gamma_L-C_L|=R_L$$

Load stability circle

Radius of circle

Center of circle



- For detailed derivation, See [2], Chapter 10 or [1], Chapter 11

Source Stability Circle (SSC)

- Setting $|\Gamma_2|=1$, we can determine all the corresponding values of Γ_s. The Γ_s also happens to fall on the locus of a circle on Smith Chart.

$$\left|s_{22}+\frac{s_{12}s_{21}\Gamma_s}{1-s_{22}\Gamma_s}\right|=1$$

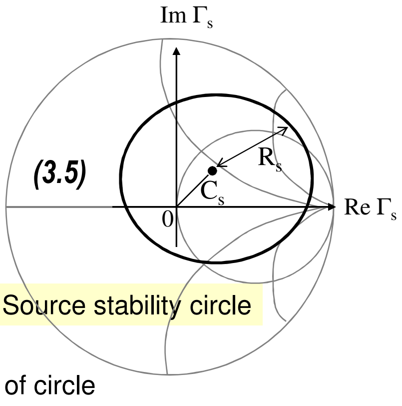
$$\Rightarrow\left|\Gamma_s-\frac{\left(s_{11}-Ds_{22}^{\ *}\right)^*}{|s_{11}|^2-|D|^2}\right|=\left|\frac{s_{12}s_{21}}{|s_{11}|^2-|D|^2}\right| \qquad \textbf{(3.5)}$$

$$\Rightarrow|\Gamma_s-C_s|=R_s$$

Source stability circle

Radius of circle

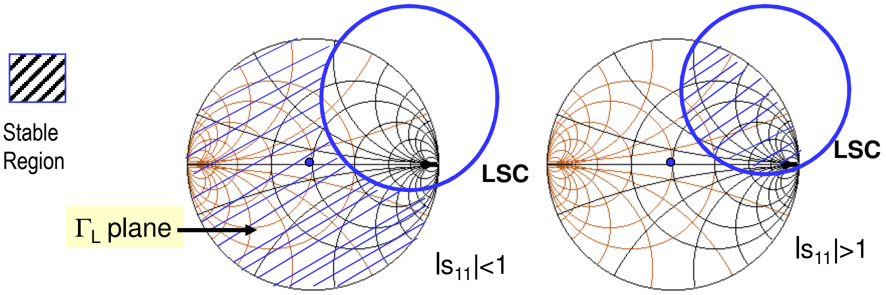
Center of circle



- For detailed derivation, See [2], chapter 10 or [1], Chapter 11

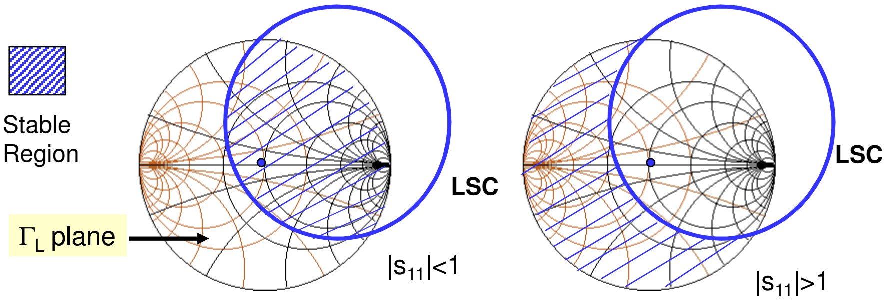
Stable Regions (1)

- The source and load stability circles only indicate the value of Γ_s and Γ_L where $|\Gamma_2| = 1$ and $|\Gamma_1| = 1$. We need more information to show the stable regions for Γ_s and Γ_L plane on the Smith Chart.
- For example for LSC, when $\Gamma_L = 0$, $|\Gamma_1| = |s_{11}|$ (See (2.2a)).
- Assume LSC does not encircle $s_{11}= 0$ point. If $|s_{11}| < 1$ then $\Gamma_L = 0$ is a stable point, else if $|s_{11}| > 1$ then $\Gamma_L=0$ is an unstable point.



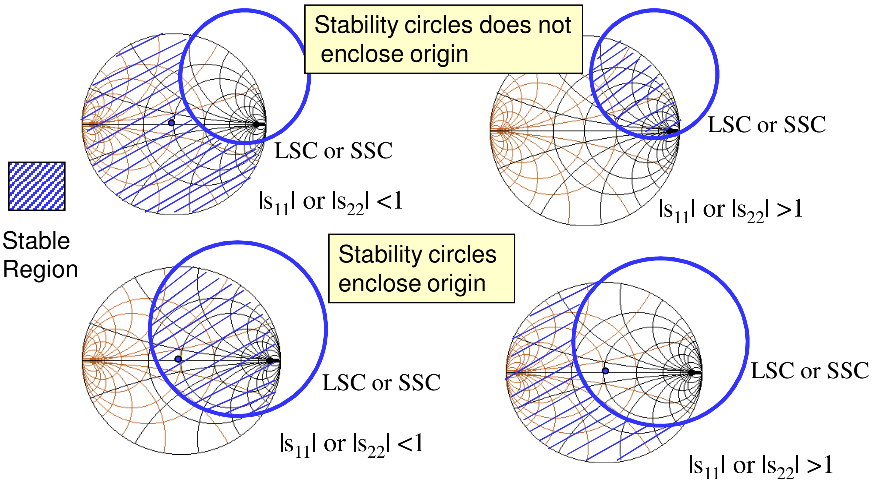
Stable Region (2)

- Now let the LSC encircles $s_{11}= 0$ point. Similarly if $|s_{11}| < 1$ then $\Gamma_L = 0$ is an stable point, else if $|s_{11}| > 1$ then $\Gamma_L= 0$ is an unstable point.
- This argument can also be applied for SSC, where we consider $|s_{22}|$ instead and the Smith Chart corresponds to Γ_s plane.



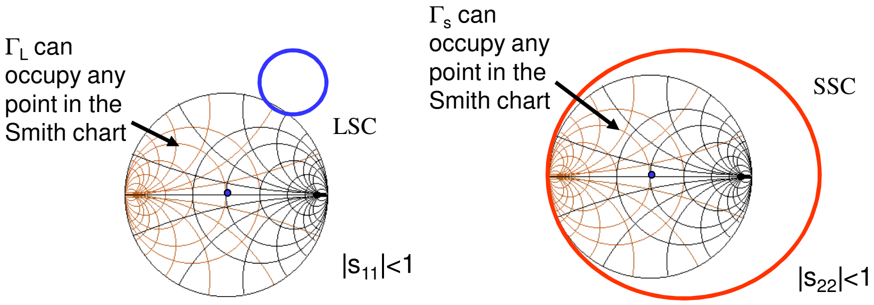
Summary for Stability Regions

- For both Source and Load reflection coefficients (Γ_s and Γ_L) :



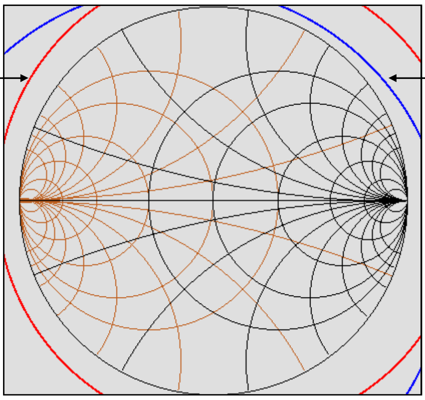
Unconditionally Stable Amplifier

- There are times when the amplifier is stable for all passive source and load impedance.
- In this case the amplifier is said to be unconditionally stable.
- Assuming $|s_{11}| < 1$ and $|s_{22}| < 1$, the stability region would look like this:



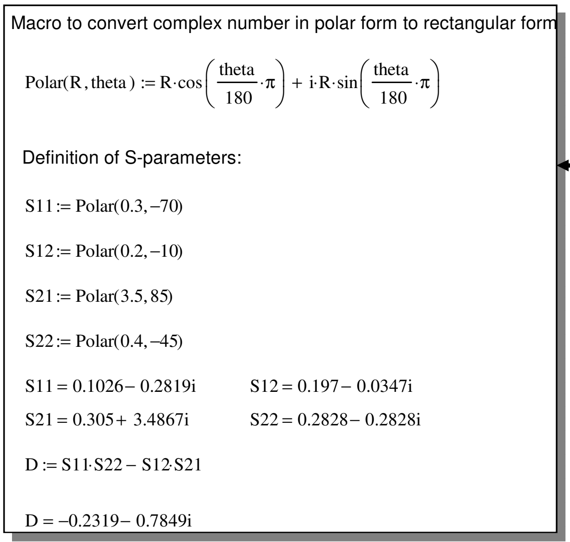
Example 3.1

- Use the S-parameters of the amplifier in Example 2.1, draw the load and source stability circles and find the stability region.



Hint:
Apply equations (3.4) and (3.5) to find the center and radius of the circles.

Example 3.1 Cont…



Macro to convert complex number in polar form to rectangular form

$$\mathrm{Polar}(R,\mathrm{theta}) := R\cdot\cos\left(\frac{\mathrm{theta}}{180}\cdot\pi\right) + i\cdot R\cdot\sin\left(\frac{\mathrm{theta}}{180}\cdot\pi\right)$$

Definition of S-parameters:

$S11 := \mathrm{Polar}(0.3,-70)$

$S12 := \mathrm{Polar}(0.2,-10)$

$S21 := \mathrm{Polar}(3.5,85)$

$S22 := \mathrm{Polar}(0.4,-45)$

$S11 = 0.1026 - 0.2819i$ $S12 = 0.197 - 0.0347i$

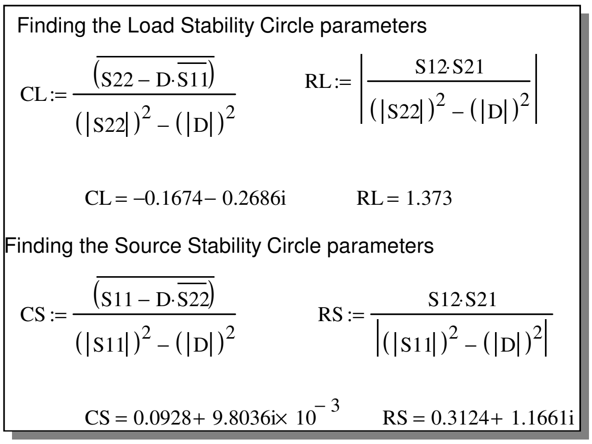
$S21 = 0.305 + 3.4867i$ $S22 = 0.2828 - 0.2828i$

$D := S11\cdot S22 - S12\cdot S21$

$D = -0.2319 - 0.7849i$

Computation Using the Software MATHCAD®

Example 3.1 Cont…



Finding the Load Stability Circle parameters

$$CL := \frac{\overline{\left(S22 - D\cdot\overline{S11}\right)}}{\left(\left|S22\right|\right)^2 - \left(\left|D\right|\right)^2} \qquad RL := \left|\frac{S12\cdot S21}{\left(\left|S22\right|\right)^2 - \left(\left|D\right|\right)^2}\right|$$

$$CL = -0.1674 - 0.2686i \qquad RL = 1.373$$

Finding the Source Stability Circle parameters

$$CS := \frac{\overline{\left(S11 - D\cdot\overline{S22}\right)}}{\left(\left|S11\right|\right)^2 - \left(\left|D\right|\right)^2} \qquad RS := \frac{S12\cdot S21}{\left|\left(\left|S11\right|\right)^2 - \left(\left|D\right|\right)^2\right|}$$

$$CS = 0.0928 + 9.8036i \times 10^{-3} \qquad RS = 0.3124 + 1.1661i$$

3.2 Test for Unconditional Stability – The Stability Factor

Stability Factor (1)

- Sometimes it is not convenient to plot the stability circles, or we just want a quick check whether an amplifier is unconditionally stable or not.
- In such condition we can compute the Stability Factor of the amplifier.
- Rollette* has come up with a factor, the K factor that tell us whether or not an amplifier (or any linear 2-port network) is unconditionally stable based on its S-parameters at a certain frequency.
- A complete derivation can be found in reference [1], [4], [5], here only the result is shown.

*Rollett J., “Stability and power gain invariants of linear two-ports”, IRE Transactions on Circuit Theory, 1962, CT-9, p. 29-32.

Stability Factor (2)

- The condition for a 2-port network to be unconditionally stable is:

$$K = \frac{1 - |s_{11}|^2 - |s_{22}|^2 + |D|^2}{2|s_{12}s_{21}|} > 1 \qquad \textbf{(3.6)}$$

$$|D| < 1$$

- Otherwise the amplifier is conditional stable or unstable at all (it is an oscillator !).
- K is also known as the Rollette Stability Factor.

Note that the K factor only tells us if an amplifier (or any linear 2-port network) is unconditionally stable. It doesn’t indicate the relative stability of 2 amplifiers which fail the test. A newer test, called the μ factor allows comparison of 2 conditionally stable amplifiers (See Appendix 2).

What if Amplifier is Unstable, or Stable Region is too Small?

- Use negative feedback to reduce amplifier gain.
- Redesign d.c. biasing, find new operating point (or Q point) that will result in more stable amplifier.
- Add some resistive loss to the circuit to improve stability.
- Use a new component with better stability.

3.3 Stabilization of Amplifier

Stabilization Methods (1)

- $|\Gamma_1| > 1$ and $|\Gamma_2| > 1$ can be written in terms of input and output impedances:

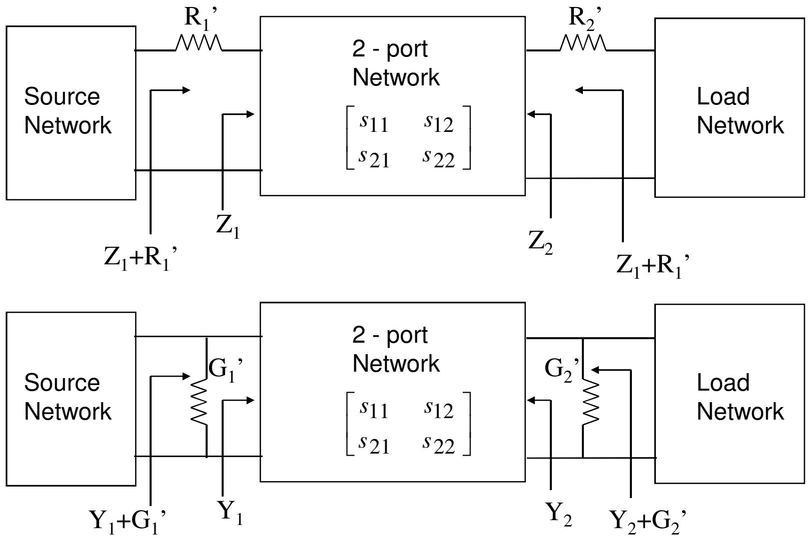
$$|\Gamma_1| = \left|\frac{Z_1 - Z_o}{Z_1 + Z_o}\right| > 1 \quad \text{and} \quad |\Gamma_2| = \left|\frac{Z_2 - Z_o}{Z_2 + Z_o}\right| > 1$$

- This implies that Re[Z_1] < 0 or Re[Z_2] < 0.

$$\Gamma_1 = \frac{(R_1 - Z_o) + jX_1}{(R_1 + Z_o) + jX_1} \Rightarrow |\Gamma_1| = \sqrt{\frac{(R_1 - Z_o)^2 + X_1^2}{(R_1 + Z_o)^2 + X_1^2}}$$

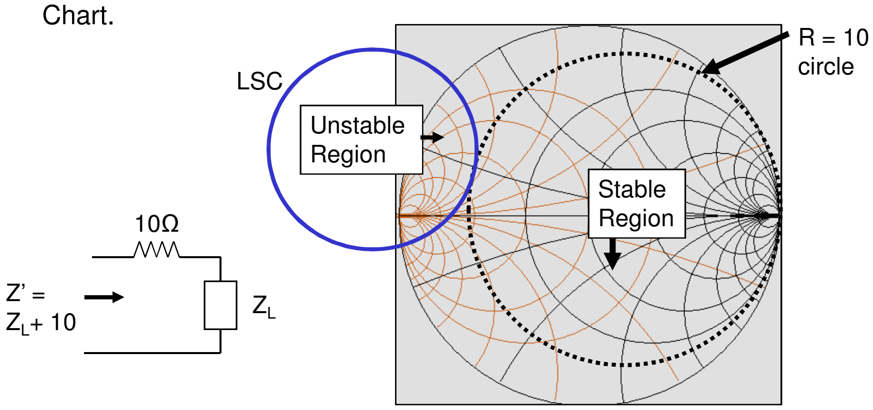
- Thus one way to stabilize an amplifier is to add a series resistance or shunt conductance to the port. This should made the real part of the impedance become positive.
- In other words we deliberately add loss to the network.

Stabilization Methods (2)



Effect of Adding Series Resistance on Smith Chart

- Suppose we have an impedance Z_L and a load stability circle (LSC). Assuming the LSC touches the R=10 circle. Thus by inserting a series resistance of 10Ω, we can limit Z_L' to the stable region on the Smith Chart.



R = 10 circle

LSC

Unstable Region

Stable Region

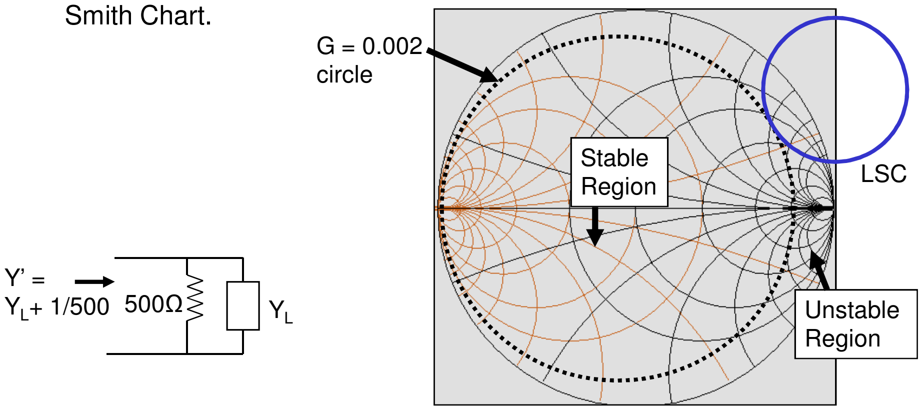
10Ω

Z' = Z_L+ 10

Z_L

Effect of Adding Shunt Resistance on Smith Chart

- Suppose we have an admittance Y_L and a load stability circle (LSC). Assuming the LSC touches the G=0.002 circle. Thus by inserting a shunt resistance of 500Ω, we can limit Y_L' to the stable region on the Smith Chart.



G = 0.002 circle

Stable Region

LSC

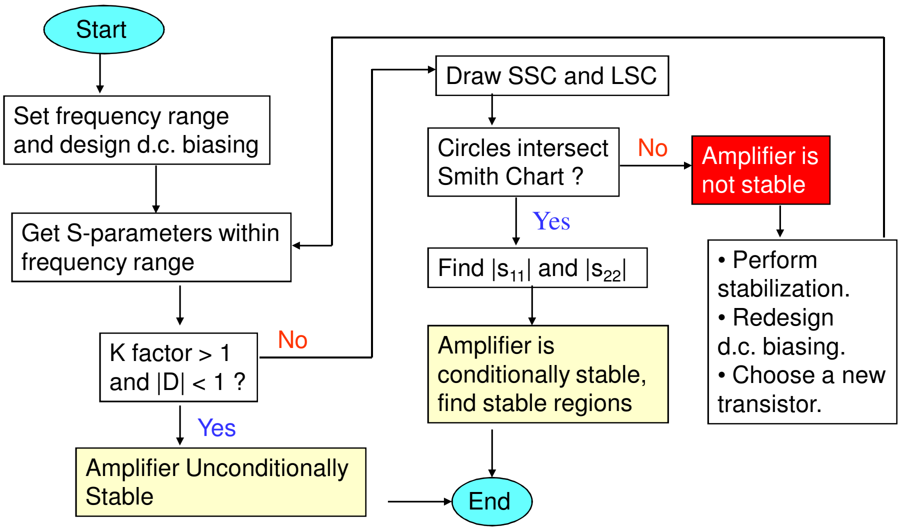
Y' = Y_L+ 1/500

500Ω

Y_L

Unstable Region

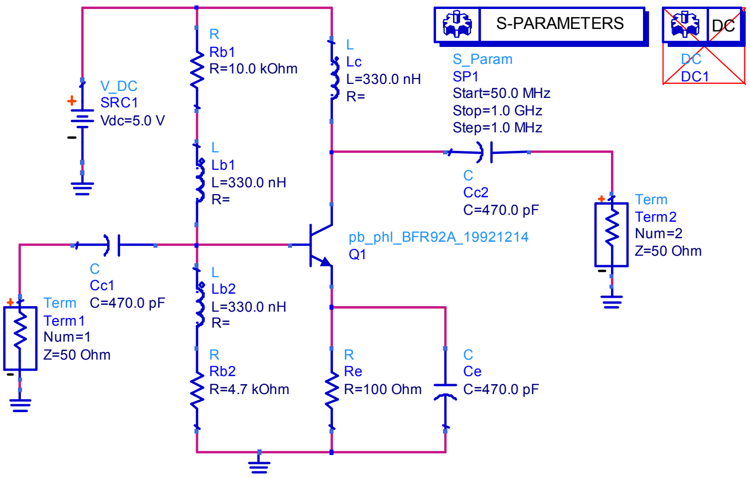
Summary for Stability Check of Single-Stage Amplifier



Chapter Summary

- Here we have learnt the important concepts of small-signal amplifier and amplifier characteristics.
- Here we have derived the three types of power gain expressions for an amplifier using S-parameters.
- We also studied how the various gains depend on either Γ_s and Γ_L (the dependency diagram).
- We have looked at the concept of oscillation and how it applies to stability analysis.
- Learnt about stability circles and how to find the stable region for source and load impedance.
- Learnt about the Rolette Stability Factor test (K) for unconditionally stable amplifier.
- Learnt about elementary stabilization methods.

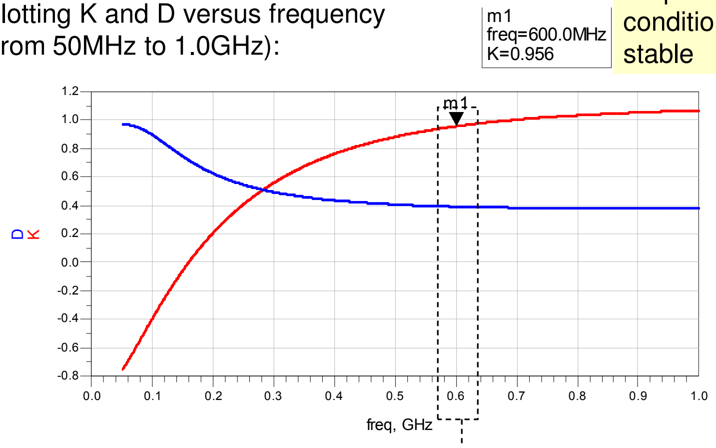
Example 3.2 - S-Parameters Measurement and Stability Analysis Using ADS Software



Example 3.2 – Stability Test

Plotting K and D versus frequency (from 50MHz to 1.0GHz):

Amplifier is conditionally stable

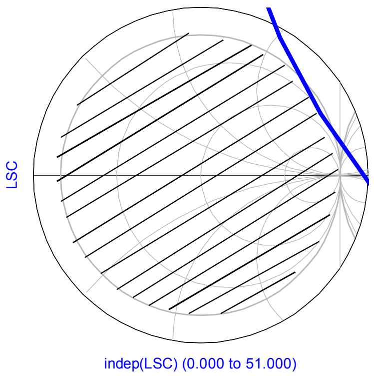


This is the frequency we are interested in

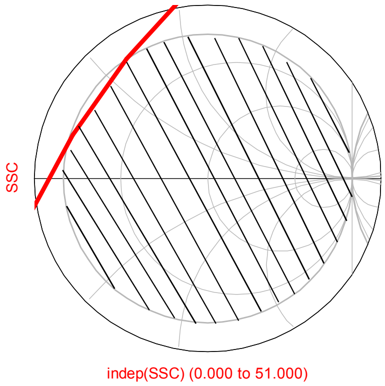
Example 3.2 - Viewing S_{11} and S_{22} at f=600MHz and Plotting Stability Circles

freq	S(1,1)	S(2,2)
600.0MHz	0.263 / -114.092	0.491 / -20.095

Since $|s_{11}| < 1$ @ 600MHz



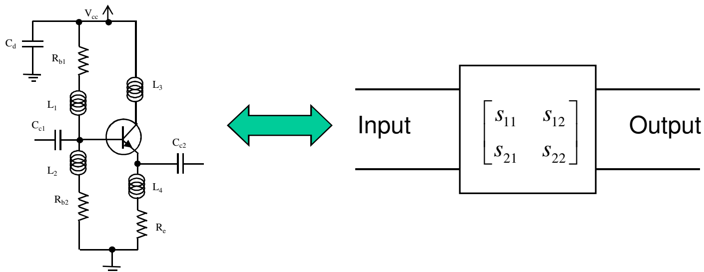
Since $|s_{22}| < 1$ @ 600MHz



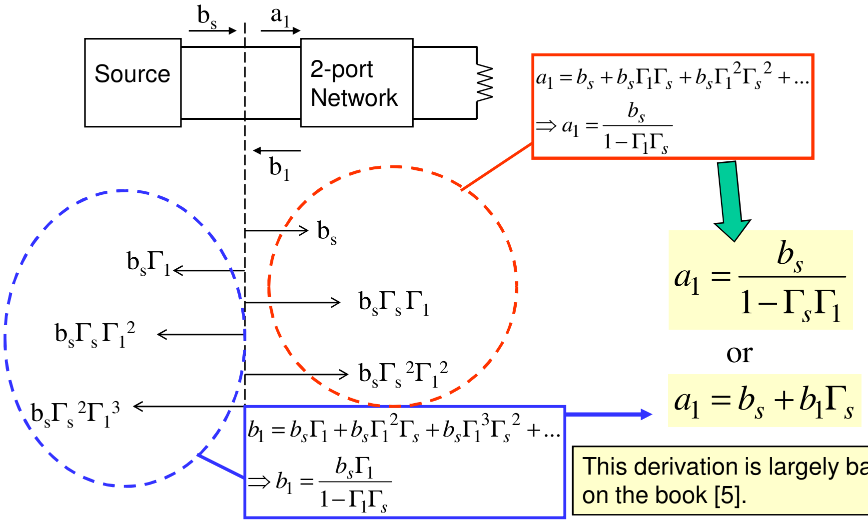
Appendix 1 – Derivation of Small-Signal Amplifier Power Gain Expressions

Derivation of Amplifier Gain Expressions in terms of S-parameters

- When the electrical signals in the amplifier is small, the active component (BJT) can be considered as linear.
- Thus the amplifier is a linear 2-port network, S-parameters can be obtained and it is modeled by an S matrix.
- The preceding gains G_P , G_A , G_T can be expressed in terms of the Γ_s , Γ_L and s_{11}, s_{12}, s_{21}, s_{22}.



Derivation of Gain Expression - 1



$$a_1 = b_s + b_s\Gamma_1\Gamma_s + b_s\Gamma_1^2\Gamma_s^2 + ...$$

$$\Rightarrow a_1 = \frac{b_s}{1-\Gamma_1\Gamma_s}$$

$$a_1 = \frac{b_s}{1-\Gamma_s\Gamma_1}$$

or

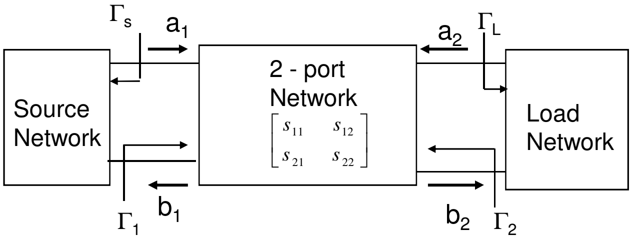
$$a_1 = b_s + b_1\Gamma_s$$

$$b_1 = b_s\Gamma_1 + b_s\Gamma_1^2\Gamma_s + b_s\Gamma_1^3\Gamma_s^2 + ...$$

$$\Rightarrow b_1 = \frac{b_s\Gamma_1}{1-\Gamma_1\Gamma_s}$$

This derivation is largely based on the book [5].

Derivation of Gain Expression - 2



From:

$$b_1 = s_{11}a_1 + s_{12}a_2$$
$$b_2 = s_{21}a_1 + s_{22}a_2$$

and

$$a_2 = \Gamma_L b_2$$
$$a_1 = \Gamma_s b_1$$

One obtains:

$$b_1 = \frac{s_{12}a_2}{1 - s_{11}\Gamma_s}$$

$$\Gamma_1 = \frac{b_1}{a_1} = \frac{s_{11} - D\Gamma_L}{1 - s_{22}\Gamma_L}$$

$$D = s_{11}s_{22} - s_{12}s_{21}$$

In a similar manner we can also obtains:

$$b_2 = \frac{s_{21}a_1}{1 - s_{22}\Gamma_L}$$

$$\Gamma_2 = \frac{b_2}{a_2} = \frac{s_{22} - D\Gamma_s}{1 - s_{11}\Gamma_s}$$

Derivation of Gain Expression - 3

Finding Transducer Power Gain G_T:

$$P_L = \tfrac{1}{2}|b_2|^2\left(1 - |\Gamma_L|^2\right)$$

$$P_A = \tfrac{1}{2}|a_1|^2\left(1 - |\Gamma_1|^2\right)\Big|_{\Gamma_s = \Gamma_1 *}$$

Only for available gain

$$a_1 = \frac{b_s}{1 - \Gamma_s\Gamma_1}\Big|_{\Gamma_s = \Gamma_1 *}$$

From slide DGE-1

$$P_A = \frac{1}{2}\frac{|b_s|^2}{1 - |\Gamma_1|^2}$$

$$G_T = \frac{\text{Load Power}}{\text{Available Soure Power}} = \frac{P_L}{P_A} = \frac{|b_2|^2}{|b_s|^2}\left(1 - |\Gamma_L|^2\right)\left(1 - |\Gamma_1|^2\right)\Big|_{\Gamma_s = \Gamma_1 *}$$

$$\Rightarrow G_T = \frac{|b_2|^2}{|b_s|^2}\left(1 - |\Gamma_L|^2\right)\left(1 - |\Gamma_s|^2\right)$$

Derivation of Gain Expression - 4

Finding Transducer Power Gain G_T Cont… :

Using

From slide DGE-2 $\frac{b_2}{a_1} = \frac{s_{21}}{1-s_{22}\Gamma_L}$

$b_s = a_1\left(1-\Gamma_1\Gamma_s\right)$ From slide DGE-1

$$\frac{b_2}{b_s} = \frac{s_{21}}{\left(1-s_{22}\Gamma_L\right)\left(1-\Gamma_1\Gamma_s\right)}$$

$$G_T = \frac{|b_2|^2}{|b_s|^2}\left(1-|\Gamma_L|^2\right)\left(1-|\Gamma_s|^2\right)$$

$$G_T = \frac{\left(1-|\Gamma_L|^2\right)|s_{21}|^2\left(1-|\Gamma_s|^2\right)}{|1-s_{22}\Gamma_L|^2|1-\Gamma_1\Gamma_s|^2}$$

$$\left(1-\Gamma_1\Gamma_s\right)\left(1-\Gamma_L s_{22}\right) = \left(1-\Gamma_2\Gamma_L\right)\left(1-\Gamma_s s_{11}\right)$$

$$G_T = \frac{\left(1-|\Gamma_L|^2\right)|s_{21}|^2\left(1-|\Gamma_s|^2\right)}{|1-s_{11}\Gamma_s|^2|1-\Gamma_2\Gamma_L|^2}$$

$$\Gamma_1 = \frac{s_{11}-D\Gamma_L}{1-s_{22}\Gamma_L}$$

$$G_T = \frac{\left(1-|\Gamma_L|^2\right)|s_{21}|^2\left(1-|\Gamma_s|^2\right)}{\left|\left(1-s_{22}\Gamma_L\right)\left(1-s_{11}\Gamma_s\right)-s_{12}s_{21}\Gamma_s\Gamma_L\right|^2}$$

Derivation of Gain Expression - 5

- The Available Power Gain G_A can be obtained from G_T when $\Gamma_L = \Gamma_2{}^*$
- Since

$$G_A = \frac{\text{Available Load Power (when output is conjugately matched)}}{\text{Available Source Power}}$$

We have…

$$G_A = G_T\Big|_{\Gamma_2=\Gamma_L*} = \left.\frac{\left(1-|\Gamma_L|^2\right)|s_{21}|^2\left(1-|\Gamma_s|^2\right)}{|1-s_{11}\Gamma_s|^2|1-\Gamma_2\Gamma_L|^2}\right|_{\Gamma_2=\Gamma_L*}$$

$$G_A = \frac{\left(1-|\Gamma_s|^2\right)|s_{21}|^2}{|1-s_{11}\Gamma_s|^2\left(1-|\Gamma_2|^2\right)}$$

Derivation of Gain Expression - 6

Finding Power Gain G_P :

$$P_L = \tfrac{1}{2}\left|b_2\right|^2\left(1-\left|\Gamma_L\right|^2\right)$$

$$P_{in} = \tfrac{1}{2}\left|a_1\right|^2\left(1-\left|\Gamma_1\right|^2\right)$$

$$\boxed{\frac{b_2}{a_1} = \frac{s_{21}}{1-s_{22}\Gamma_L}}$$

From slide DGE-2

$$G_P = \frac{\text{Load Power}}{\text{Source Power}} = \frac{P_L}{P_{in}} = \frac{\left|s_{21}\right|^2\left(1-\left|\Gamma_L\right|^2\right)}{\left|1-s_{22}\Gamma_L\right|^2\left(1-\left|\Gamma_1\right|^2\right)}$$

Appendix 2 – The μ Stability Test

The μ Stability Test

- The Roulette Stability Test consist of 2 separate tests (the K and D values).
- This makes it difficult to compare the relative stability of 2 conditionally stable amplifiers.
- A later development combines the 2 tests into one, known as the μ factor. Larger value indicates greater stability.
- For an amplifier to be unconditionally stable, it is necessary that:

$$\mu_1 = \frac{1 - |s_{22}|^2}{|s_{11} - D(s_{22}*)| + |s_{21}s_{12}|} > 1$$

Edwards, M. L., and J. H. Sinsky, “A new criterion for linear two-port stability using A single geometrically derived parameter”, IEEE Trans. On Microwave Theory and Techniques, Dec 1992.