3B. RF/Microwave Filters

References

- [1] R. E. Collin, "Foundations for microwave engineering", 2nd Edition 1992, McGraw-Hill.
- [2] D. M. Pozar, "Microwave engineering", 2nd Edition 1998, John Wiley & Sons.* (3rd Edition 2005, John-Wiley & Sons is now available)
- Other more advanced references:
- [3] W. Chen (editor), "The circuits and filters handbook", 1995, CRC Press.*
- [4] I. Hunter, "Theory and design of microwave filters", 2001, The Instutitution of Electrical Engineers.*
- [5] G. Matthaei, L. Young, E.M.T. Jones, "Microwave filters, impedance-matching networks, and coupling structures", 1980, Artech House.*
- [6] F. F. Kuo, "Network analysis and synthesis", 2nd edition 1966, John-Wiley & Sons.

* Recommended

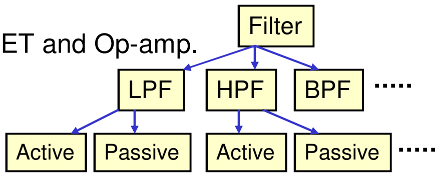
1.0 Basic Filter Theory

Introduction

- An ideal filter is a linear 2-port network that provides perfect transmission of signal for frequencies in a certain passband region, infinite attenuation for frequencies in the stopband region and a linear phase response in the passband (to reduce signal distortion).
- The goal of filter design is to approximate the ideal requirements within acceptable tolerance with circuits or systems consisting of real components.

Categorization of Filters

- Low-pass filter (LPF), High-pass filter (HPF), Bandpass filter (BPF), Bandstop filter (BSF), arbitrary type etc.
- In each category, the filter can be further divided into active and passive types.
- In active filter, there can be amplification of the of the signal power in the passband region, passive filter do not provide power amplification in the passband.
- Filter used in electronics can be constructed from resistors, inductors, capacitors, transmission line sections and resonating structures (e.g. piezoelectric crystal, Surface Acoustic Wave (SAW) devices, and also mechanical resonators etc.).
- Active filter may contain transistor, FET and Op-amp.

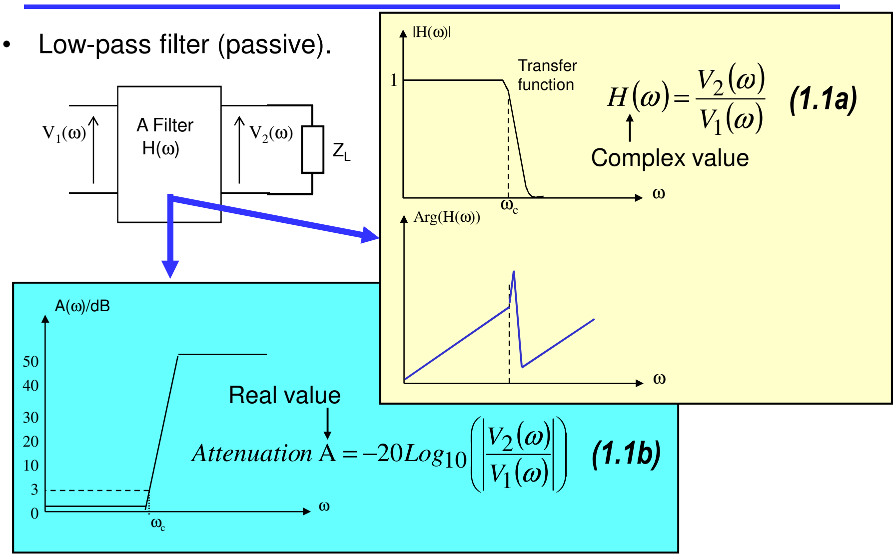


Filter's Frequency Response (1)

- Frequency response implies the behavior of the filter with respect to steady-state sinusoidal excitation (e.g. energizing the filter with sine voltage or current source and observing its output).
- There are various approaches to displaying the frequency response:
  - Transfer function $H(\omega)$ (the traditional approach).
  - Attenuation factor $A(\omega)$.
  - S-parameters, e.g. $s_{21}(\omega)$.
  - Others, such as ABCD parameters etc.

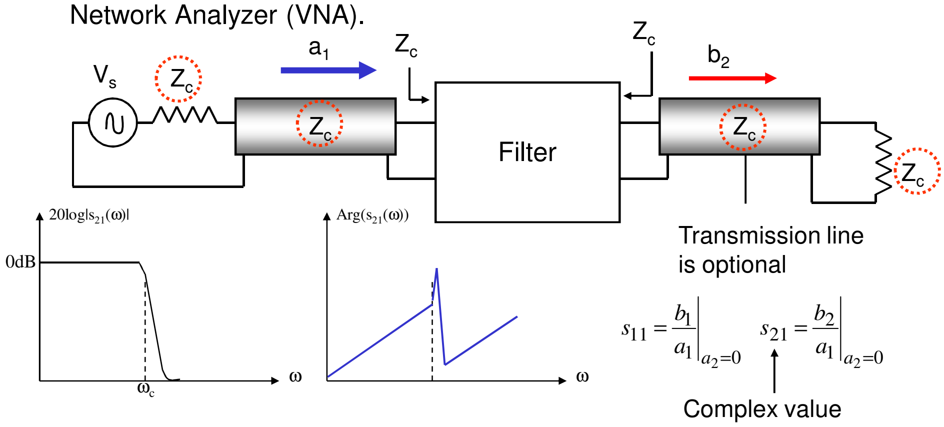
Filter Frequency Response (2)

- Low-pass filter (passive).



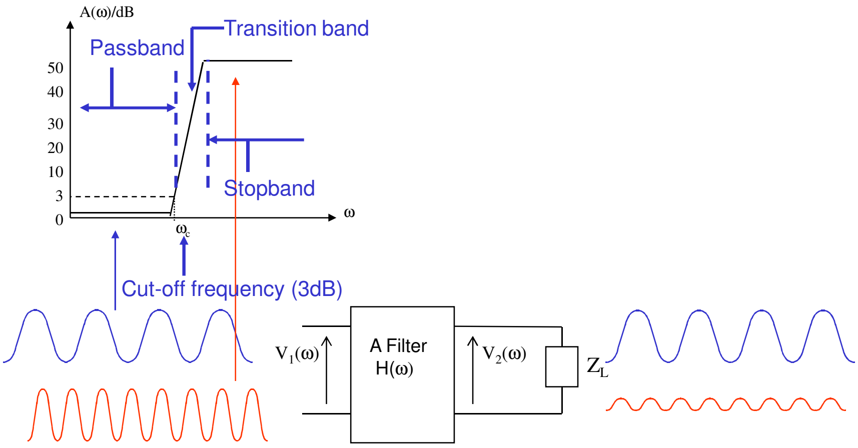
Filter Frequency Response (3)

- Low-pass filter (passive) continued...
- For impedance matched system, using s_{21} to observe the filter response is more convenient, as this can be easily measured using Vector Network Analyzer (VNA).



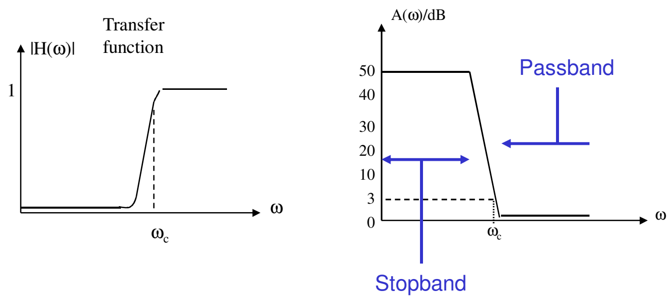
Filter Frequency Response (4)

- Low-pass filter (passive) continued...



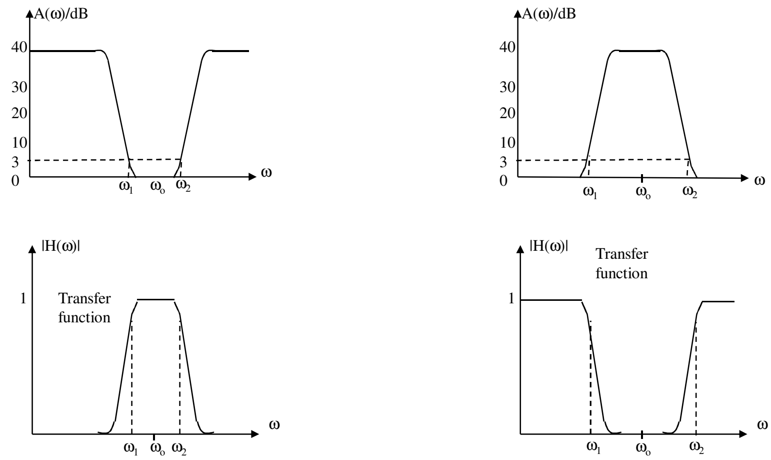
Filter Frequency Response (5)

- High-pass filter (passive).



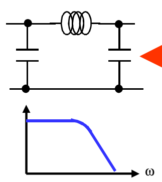
Filter Frequency Response (6)

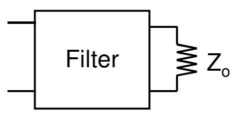
- Band-pass filter (passive). Band-stop filter.

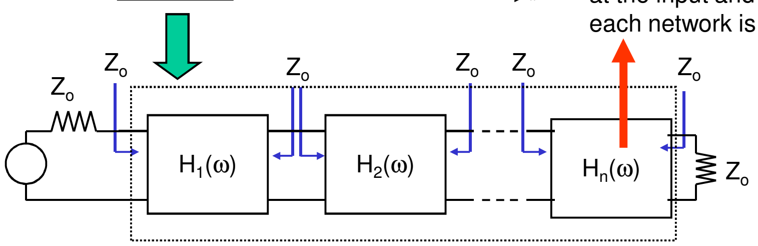


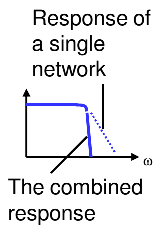
Basic Filter Synthesis Approaches (1)

- Image Parameter Method (See [4] and [2]).
  - Consider a filter to be a cascade of linear 2-port networks.
  - Synthesize or realize each 2-port network, so that the combine effect gives the required frequency response.
  - The 'image' impedance seen at the input and output of each network is maintained.





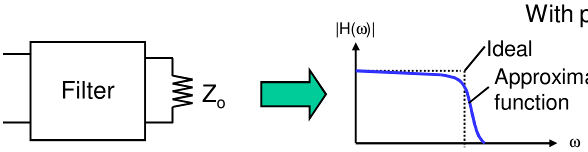




Basic Filter Synthesis Approaches (2)

- Insertion Loss Method (See [2]).

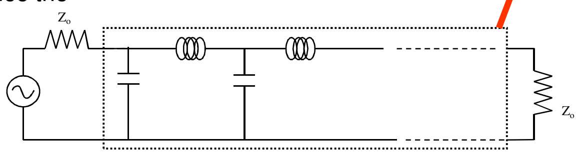
Approximate ideal filter response
With polynomial function:



$$H(s) = K\frac{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_o}{s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_o}$$

We can also use Attenuation Factor or $|s_{21}|$ for this.

Use RCLM circuit synthesis theorem ([3], [6]) to come up with a resistive terminated LC network that can produce the approximate response.



Our Scope

- Only concentrate on passive LC and stripline filters.
- Filter synthesis using the Insertion Loss Method (ILM). The Image Parameter Method (IPM) is more efficient and suitable for simple filter designs, but has the disadvantage that arbitrary frequency response cannot be incorporated into the design.

2.0 Passive LC Filter Synthesis Using Insertion Loss Method

Insertion Loss Method (ILM)

- The insertion loss method (ILM) allows a systematic way to design and synthesize a filter with various frequency response.
- ILM method also allows filter performance to be improved in a straightforward manner, at the expense of a 'higher order' filter.
- A rational polynomial function is used to approximate the ideal $|H(\omega)|$, $A(\omega)$ or $|s_{21}(\omega)|$.
- Phase information is totally ignored.
- Ignoring phase simplified the actual synthesis method. An LC network is then derived that will produce this approximated response.
- Here we will use $A(\omega)$ following [2]. The attenuation $A(\omega)$ can be cast into power attenuation ratio, called the Power Loss Ratio, P_{LR}, which is related to $A(\omega)^2$.

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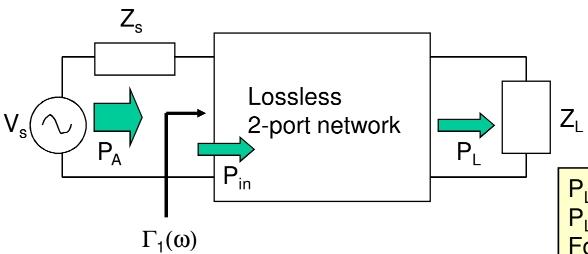
Extra

More on ILM

- There is a historical reason why phase information is ignored. Original filter synthesis methods are developed in the 1920s-60s, for voice communication. Human ear is insensitive to phase distortion, thus only magnitude response (e.g. |H(ω)|, A(ω)) is considered.
- Modern filter synthesis can optimize a circuit to meet both magnitude and phase requirements. This is usually done using computer optimization procedures with 'goal functions'.

Power Loss Ratio (P_{LR})

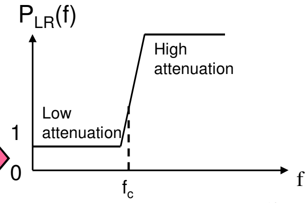


P_{LR} large, high attenuation
P_{LR} close to 1, low attenuation
For example, a low-pass filter response is shown below:

$$P_{LR} = \frac{\text{Power available from source network}}{\text{Power delivered to Load}}$$

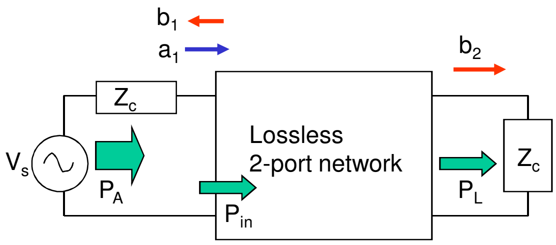
$$= \frac{P_{inc}}{P_{Load}} = \frac{P_A}{P_A\left[1-\left|\Gamma_1(\omega)\right|^2\right]} = \frac{1}{1-\left|\Gamma_1(\omega)\right|^2} \quad \textbf{(2.1a)}$$

Low-Pass filter P_{LR}



P_{LR} and s_{21}

- In terms of incident and reflected waves, assuming $Z_L = Z_s = Z_C$.



$$P_{LR} = \frac{P_A}{P_L} = \frac{\frac{1}{2}|a_1|^2}{\frac{1}{2}|b_2|^2} = \left|\frac{a_1}{b_2}\right|^2$$

$$P_{LR} = \frac{1}{|s_{21}|^2} \quad \textbf{(2.1b)}$$

P_{LR} for Low-Pass Filter (LPF)

- Since $|\Gamma_1(\omega)|^2$ is an <u>even</u> function of ω, it can be written in terms of ω^2 as:

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)} \quad \textbf{(2.2)}$$

This is also known as Characteristic Polynomial

- P_{LR} can be expressed as:

$$P_{LR} = \frac{1}{1 - |\Gamma_1(\omega)|^2} = \frac{1}{1 - \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

$$P_{LR} = 1 + [P(\omega)]^2 \quad \textbf{(2.3a)}$$

$$[P(\omega)]^2 = \frac{M(\omega^2)}{N(\omega^2)} \quad \textbf{(2.3b)}$$

$P(\omega)P(\omega)$

- Various type of polynomial functions in ω can be used for $P(\omega)$. The requirement is $P(\omega)$ must be either odd or even function. Among the classical polynomial functions are:
  - Maximally flat or Butterworth functions.
  - Equal ripple or Chebyshev functions.
  - Elliptic function.
  - Many, many more.

The characteristics we need from $[P(\omega)]^2$ for LPF:
- $[P(\omega)]^2 \rightarrow 0$ for $\omega < \omega_c$
- $[P(\omega)]^2 >> 1$ for $\omega >> \omega_c$

Characteristic Polynomial Functions

- Maximally flat or Butterworth: $P(\omega)=\left(\frac{\omega}{\omega_c}\right)^N$ **(2.4a)**

N = order of the Characteristic Polynomial P(ω)

- Equal ripple or Chebyshev: $P(\omega)=\varepsilon C_N(\omega)$, $\varepsilon =$ ripple factor

$$C_N(\omega)=\begin{cases}C_0(\omega)=1\\ C_1(\omega)=\omega\\ C_n(\omega)=2\omega C_{n-1}(\omega)-C_{n-2}(\omega)\ ,n\geq 2\end{cases} \quad \textbf{(2.4b)}$$

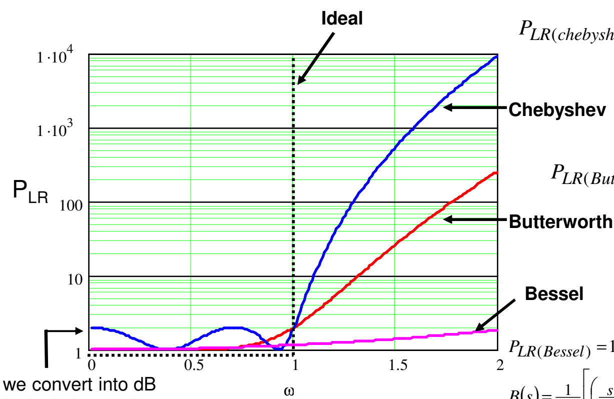
- Bessel [6] or linear phase: $[P(\omega)]^2 = B(j\omega)B(-j\omega)-1$

$$B_N(s)=\begin{cases}B_0(s)=1\\ B_1(s)=s+1\\ B_n(s)=(2s-1)B_{n-1}(s)+s^2B_{n-2}(s)\ ,n\geq 2\end{cases} \quad \textbf{(2.4c)}$$

For other types of polynomial functions, please refer to reference [3] and [6].

Examples of P_{LR} for Low-Pass Filter (1)

- P_{LR} of low pass filter using 4th order polynomial functions (N=4) - Butterworth, Chebyshev (ripple factor =1) and Bessel. Normalized to ω_c = 1 rad/s, k=1.



$$P_{LR(chebyshev)} = 1+k^2\left[8\left(\frac{\omega}{\omega_c}\right)^2-4\left(\frac{\omega}{\omega_c}\right)+1\right]^2$$

k=1

$$P_{LR(Butterworth)} = 1+k^2\left[\left(\frac{\omega}{\omega_c}\right)^4\right]^2$$

$$P_{LR(Bessel)} = 1+k^2[B(j\omega)B(-j\omega)-1]$$

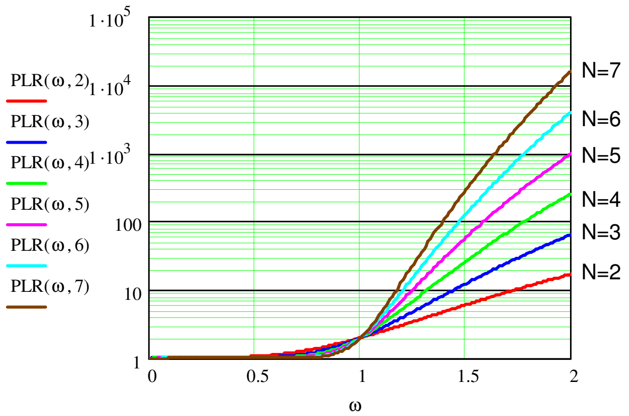
$$B(s)=\frac{1}{105}\left[\left(\frac{s}{\omega_c}\right)^4+10\left(\frac{s}{\omega_c}\right)^3+45\left(\frac{s}{\omega_c}\right)^2+105\left(\frac{s}{\omega_c}\right)+105\right]$$

If we convert into dB this ripple is equal to 3 dB

Examples of P_{LR} for Low-Pass Filter (2)

- P_{LR} of low pass filter using Butterworth characteristic polynomial, normalized to ω_c = 1 rad/s, k=1.

$$P_{LR(Butterworth)} = 1 + k^2\left[\left(\frac{\omega}{\omega_c}\right)^N\right]^2$$



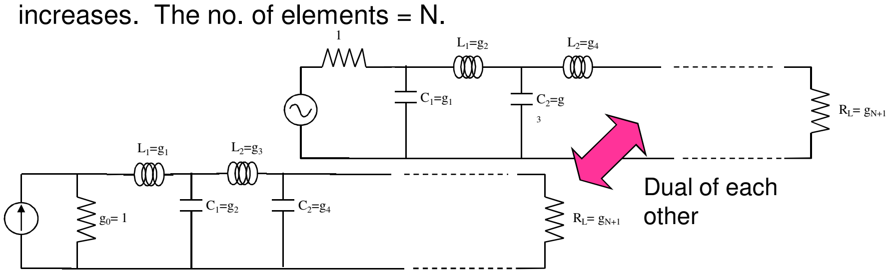
<u>Conclusion</u>: The type of polynomial function and the order determine the Attenuation rate in the stopband.

Characteristics of Low-Pass Filters Using Various Polynomial Functions

- **Butterworth**: Moderately linear phase response, slow cut-off, smooth attenuation in passband.
- **Chebyshev**: Bad phase response, rapid cut-off for similar order, contains ripple in passband. May have impedance mismatch for N even.
- **Bessel**: Good phase response, linear. Very slow cut-off. Smooth amplitude response in passband.

Low-Pass Prototype Design (1)

- A lossless linear, passive, reciprocal network that can produce the insertion loss profile for Low-Pass Filter is the LC ladder network.
- Many researchers have tabulated the values for the L and C for the Low-Pass Filter with cut-off frequency ω_c = 1 Rad/s, that works with source and load impedance $Z_s = Z_L$ = 1 Ohm.
- This Low-Pass Filter is known as the Low-Pass Prototype (LPP).
- As the order N of the polynomial P increases, the required element also increases. The no. of elements = N.



Low-Pass Prototype Design (2)

- The LPP is the 'building block' from which real filters may be constructed.
- Various transformations may be used to convert it into a high-pass, band-pass or other filter of arbitrary center frequency and bandwidth.
- The following slides show some sample tables for designing LPP for Butterworth and Chebyshev amplitude response of P_{LR}.
- See Chapter 3 of Hunter [4], on how the LPP circuits and the tables can be derived.

Table for Butterworth LPP Design

N	g1	g2	g3	g4	g5	g6	g7	g8	g9
1	2.0000	1.0000							
2	1.4142	1.4142	1.0000						
3	1.0000	2.0000	1.0000	1.0000					
4	0.7654	1.8478	1.8478	0.7654	1.0000				
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000			
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000		
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000	
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000

Taken from Chapter 8, Pozar [2].

See Example 2.1 in the following slides on how the constant values g_1, g_2, g_3…etc. are obtained.

Table for Chebyshev LPP Design

- Ripple factor $20\log_{10}\varepsilon = 0.5\text{dB}$

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7
1	0.6986	1.0000					
2	1.4029	0.7071	1.9841				
3	1.5963	1.0967	1.5963	1.0000			
4	1.6703	1.1926	2.3661	0.8419	1.9841		
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000	
6	1.7254	1.2479	2.6064	1.3137	2.4578	0.8696	1.9841

- Ripple factor $20\log_{10}\varepsilon = 3.0\text{dB}$

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7
1	1.9953	1.0000					
2	3.1013	0.5339	5.8095				
3	3.3487	0.7117	3.3487	1.0000			
4	3.4389	0.7483	4.3471	0.5920	5.8095		
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000	
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095

Table for Maximally-Flat Time Delay LPP Design

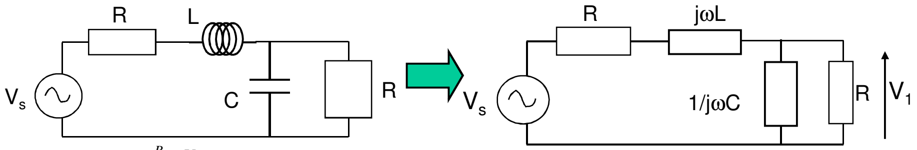
N	g1	g2	g3	g4	g5	g6	g7	g8	g9
1	2.0000	1.0000							
2	1.5774	0.4226	1.0000						
3	1.2550	0.5528	0.1922	1.0000					
4	1.0598	0.5116	0.3181	0.1104	1.0000				
5	0.9303	0.4577	0.3312	0.2090	0.0718	1.0000			
6	0.8377	0.4116	0.3158	0.2364	0.1480	0.0505	1.0000		
7	0.7677	0.3744	0.2944	0.2378	0.1778	0.1104	0.0375	1.0000	
8	0.7125	0.3446	0.2735	0.2297	0.1867	0.1387	0.0855	0.0289	1.0000

Taken from Chapter 8, Pozar [2].

Example 2.1 - Finding the Constants for LPP Design (1)

Extra

Consider a simple case of 2nd order Low-Pass Filter:



$$V_1(\omega)=\frac{\frac{R}{1+j\omega RC}V_s}{R+j\omega L+\frac{R}{1+j\omega RC}}=\frac{RV_s}{R+(R+j\omega L)(1+j\omega RC)}=\frac{RV_s}{2R-\omega^2 RLC+j\omega\left(L+R^2C\right)}$$

Thus $P_L(\omega)=\frac{1}{2R}|V_1(\omega)|^2=\frac{|V_s|^2 R}{2\left[\left(2-\omega^2 LC\right)^2R^2+\omega^2\left(L+R^2C\right)^2\right]}$ and $P_A=\frac{1}{8R}|V_s|^2$

Therefore we can compute the power loss ratio as:

$$P_{LR}(\omega)=\frac{P_A}{P_L(\omega)}=\frac{\frac{|V_s|^2}{8R}}{\frac{|V_s|^2 R}{2\left[\left(2-\omega^2 LC\right)^2R^2+\omega^2\left(L+R^2C\right)^2\right]}}=\frac{1}{8R^2}\left[2R^2\left(2-\omega^2 LC\right)^2+2\left(L+R^2C\right)^2\omega^2\right]$$

$$=1+\left[\left(\frac{1}{4R^2}\left(L+R^2C\right)^2-LC\right)\omega^2+\left(\frac{LC}{2}\right)^2\omega^4\right]$$

$[P(\omega)]^2$

Extra

Example 2.1 - Finding the Constants for LPP Design (2)

P_{LR} can be written in terms of polynomial of ω^2:

$$P_{LR}(\omega)=1+\left[\left(\frac{1}{4R^2}\left(L+R^2C\right)^2-LC\right)\omega^2+\left(\frac{LC}{2}\right)^2\omega^4\right]=1+\left[a_1\omega^2+a_2\omega^4\right] \quad \text{(E1.1)}$$

For Butterworth response with k=1, ω_c = 1:

$$P_{LR(Butterworth)}=1+\left[\omega^2\right]^2=1+\omega^4=1+0\cdot\omega^2+1\cdot\omega^4 \quad \text{(E1.2)}$$

Comparing equation (E1.1) and (E1.2):

$$a_2=1\Rightarrow\frac{LC}{2}=1\Rightarrow LC=2 \quad \text{(E1.3)}$$

$$a_1=0\Rightarrow\frac{1}{4R^2}\left(L+R^2C\right)^2-LC=0$$

$$\Rightarrow\sqrt{LC}=\frac{1}{2R}\left(L+R^2C\right) \quad \text{(E1.4)}$$

Setting R=1 for Low-Pass Prototype (LPP):

$R=1$ Thus from equation (E1.4):

$$\sqrt{LC}=\tfrac{1}{2}(L+C)^2\Rightarrow L^2+C^2-2LC=0$$

$$\Rightarrow(L-C)^2=0$$

$$\Rightarrow L=C$$

Using (E1.3)

$$LC=2\Rightarrow C^2=2$$

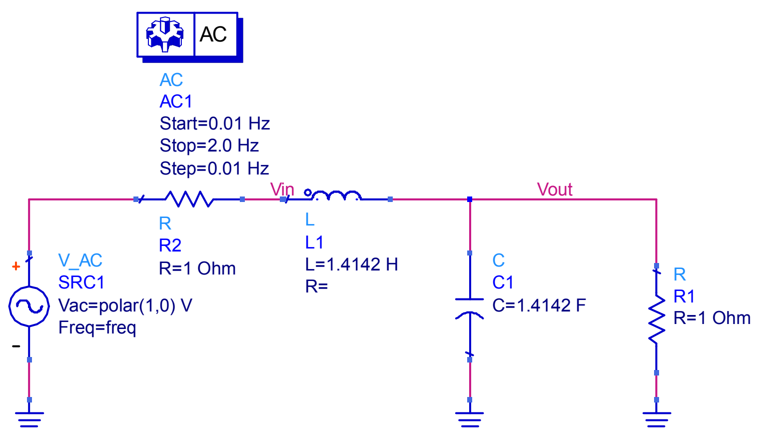
$$\Rightarrow C=\sqrt{2}\cong1.4142$$

$$L=C\cong1.4142$$

Compare this result with N=2 in the table for LPP Butterworth response. This direct 'brute force' approach can be extended to N=3, 4, 5…

Extra

Example 2.1 – Verification (1)

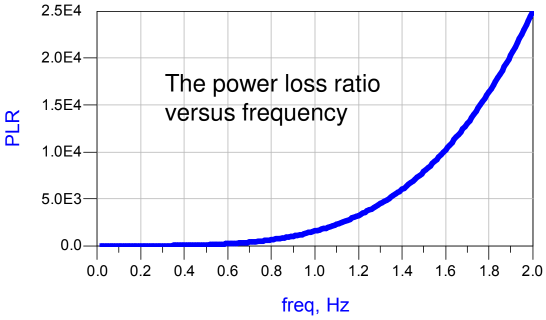


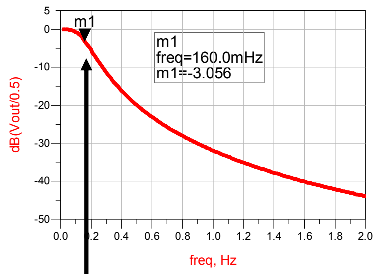
Example 2.1 – Verification (2)

Extra

Eqn PA=1/8   Eqn PL=0.5*mag(Vout)*mag(Vout)

Eqn PLR=PA/PL



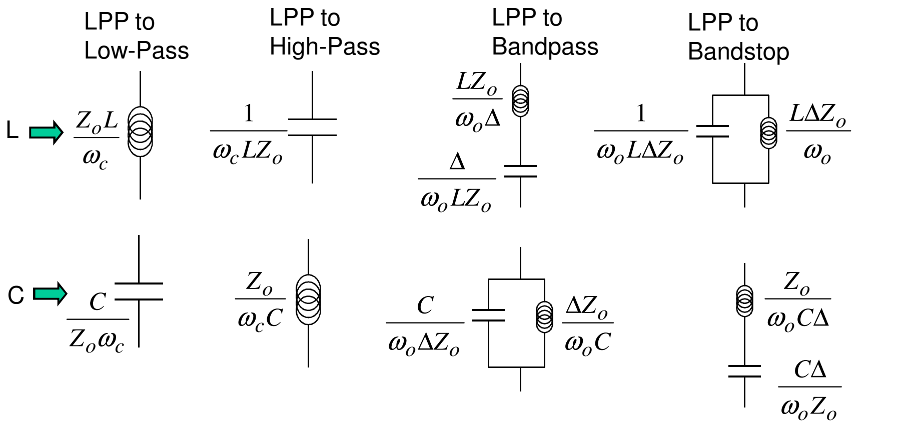


-3dB at 160mHz (miliHertz!!), which is equivalent to 1 rad/s

Impedance Denormalization and Frequency Transformation of LPP (1)

- Once the LPP filter is designed, the cut-off frequency ω_c can be transformed to other frequencies.
- Furthermore the LPP can be mapped to other filter types such as high-pass, bandpass and bandstop (see [2] and [3] for the derivation and theories).
- This frequency scaling and transformation entails changing the value and configuration of the elements of the LPP.
- Finally the impedance presented by the filter at the operating frequency can also be scaled, from unity to other values, this is called impedance denormalization.
- Let Z_o be the new system impedance value. The following slide summarizes the various transformation from the LPP filter.

Impedance Denormalization and Frequency Transformation of LPP (2)



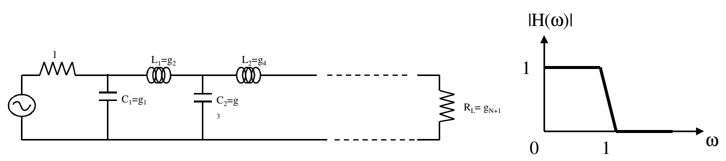
$$\omega_o = \frac{\omega_1 + \omega_2}{2} \text{ or } \sqrt{\omega_1 \omega_2} \quad \textit{(2.5a)}$$

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_o} \quad \textit{(2.5b)}$$

Note that inductor always multiply with Z_o while capacitor divide with Z_o

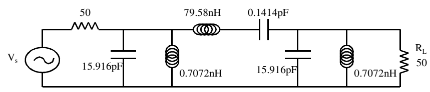
Summary of Passive LC Filter Design Flow Using ILM Method (1)

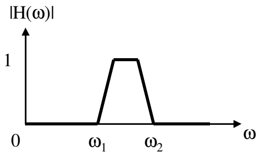
- **Step 1** - From the requirements, determine the order and type of approximation functions to used.
  - **Insertion loss (dB) in passband ?**
  - **Attenuation (dB) in stopband ?**
  - **Cut-off rate (dB/decade) in transition band ?**
  - **Tolerable ripple?**
  - **Linearity of phase?**
- **Step 2** - Design the normalized low-pass prototype (LPP) using L and C elements.



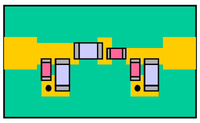
Summary of Passive Filter Design Flow Using ILM Method (2)

- **Step 3** - Perform frequency scaling and denormalize the impedance.

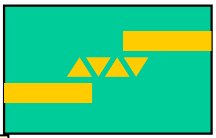




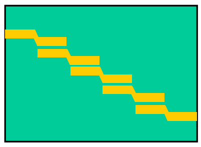
- **Step 4** - Choose suitable lumped components, or transform the lumped circuit design into distributed realization.



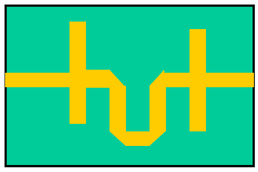
All uses microstrip stripline circuit



See Ref. [4]



See Ref. [2]



See Ref. [3]

Filter vs Impedance Transformation Network

- If we ponder carefully, the sharp observer will notice that the filter can be considered as a class of impedance transformation network.
- In the passband, the load is matched to the source network, much like a filter.
- In the stopband, the load impedance is highly mismatched from the source impedance.
- However, the procedure described here only applies to the case when both load and source impedance are equal and real.

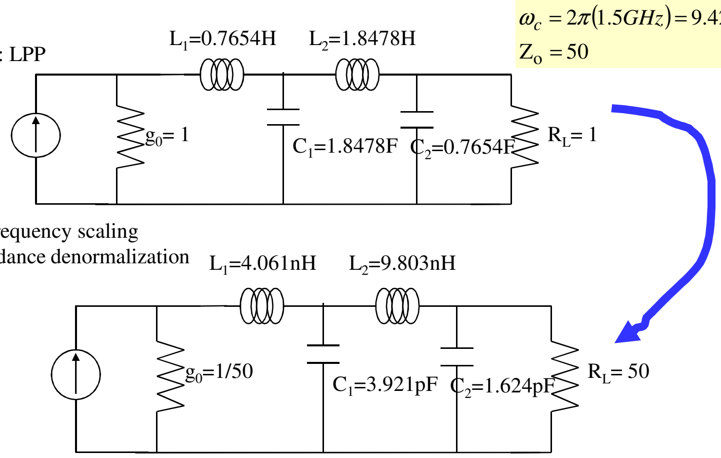
Example 2.2A – LPF Design: Butterworth Response

- Design a 4th order Butterworth Low-Pass Filter. $R_s = R_L = 50$Ohm, $f_c = 1.5$GHz.

$\omega_c = 2\pi(1.5GHz) = 9.4248\times 10^9$ rad/s
$Z_o = 50$

Step 1&2: LPP

Step 3: Frequency scaling and impedance denormalization



$$R = Z_o R_n \qquad L = Z_o \frac{L_n}{\omega_c} \qquad C = \frac{C_n}{Z_o \omega_c}$$

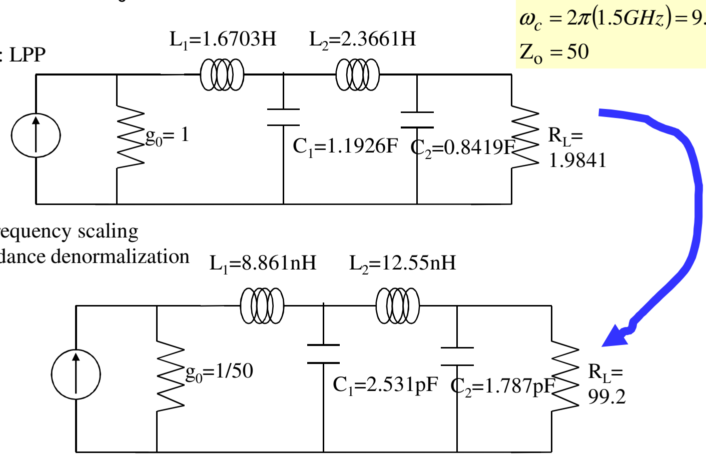
Example 2.2B – LPF Design: Chebyshev Response

- Design a 4th order Chebyshev Low-Pass Filter, 0.5dB ripple factor. $R_s = 50$Ohm, $f_c = 1.5$GHz.

$\omega_c = 2\pi(1.5GHz) = 9.4248\times 10^9$ rad/s
$Z_o = 50$

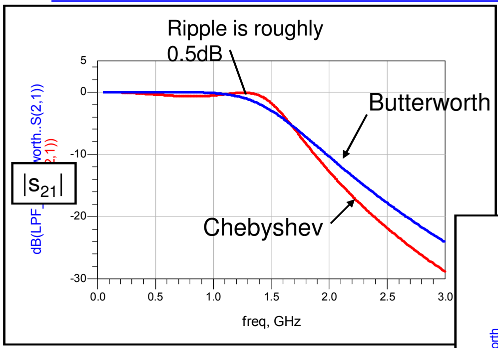
Step 1&2: LPP

Step 3: Frequency scaling and impedance denormalization

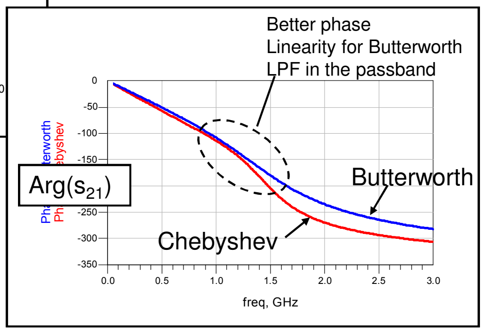


$$R = Z_o R_n \qquad L = Z_o \frac{L_n}{\omega_c} \qquad C = \frac{C_n}{Z_o \omega_c}$$

Example 2.2 Cont...



Computer simulation result
Using AC analysis (ADS2003C)



Note: Equation used in Data Display of ADS2003C to obtain continuous phase display with built-in function phase( ).

Eqn Phase_chebyshev = if (phase(S(2,1))<0) then phase(S(2,1)) else (phase(S(2,1))-360)

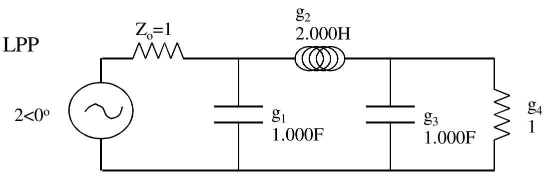
Example 2.3: BPF Design

- Design a bandpass filter with Butterworth (maximally flat) response.
- N = 3.
- Center frequency f_o = 1.5GHz.
- 3dB Bandwidth = 200MHz or f_1=1.4GHz, f_2=1.6GHz.
- Impedance = 50Ω.

Example 2.3 Cont…

- From table, design the Low-Pass prototype (LPP) for 3rd order Butterworth response, ω_c=1.

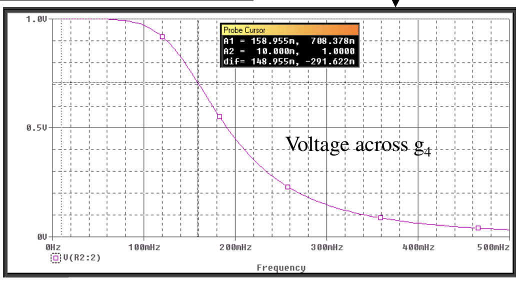
Step 1&2: LPP



$$\omega_c = 2\pi f_c = 1$$

$$\Rightarrow f_c = \frac{1}{2\pi} = 0.1592 \text{ Hz}$$

Simulated result using PSPICE



Example 2.3 Cont…

- LPP to bandpass transformation.
- Impedance denormalization.

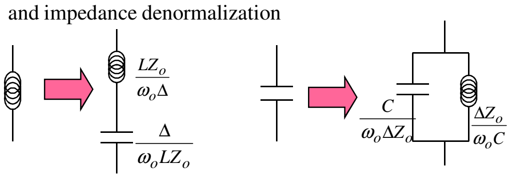
Step 3: Frequency scaling and impedance denormalization

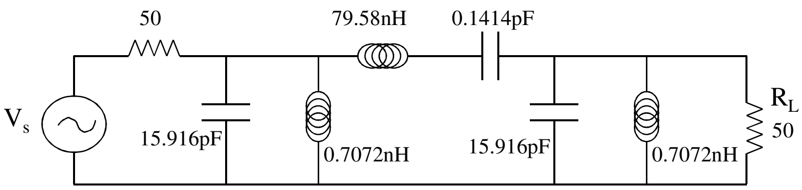
$$\omega_1 = 2\pi(1.4GHz)$$

$$\omega_2 = 2\pi(1.6GHz)$$

$$f_o = \sqrt{f_1 f_2} = 1.497GHz$$

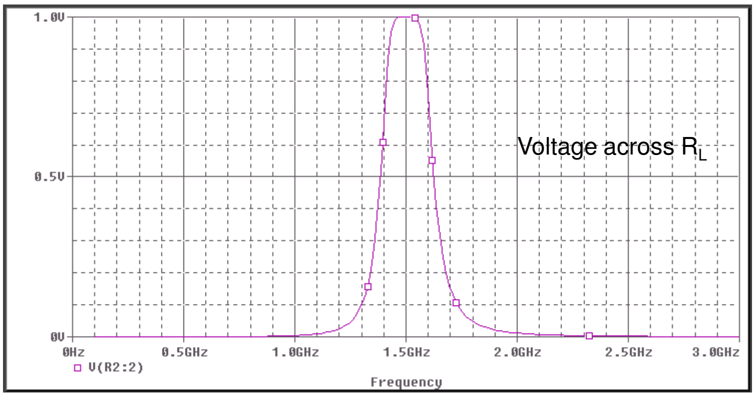
$$\Delta = \frac{\omega_2 - \omega_1}{\omega_o} = 0.133$$





Example 2.3 Cont…

- Simulated result using PSPICE:

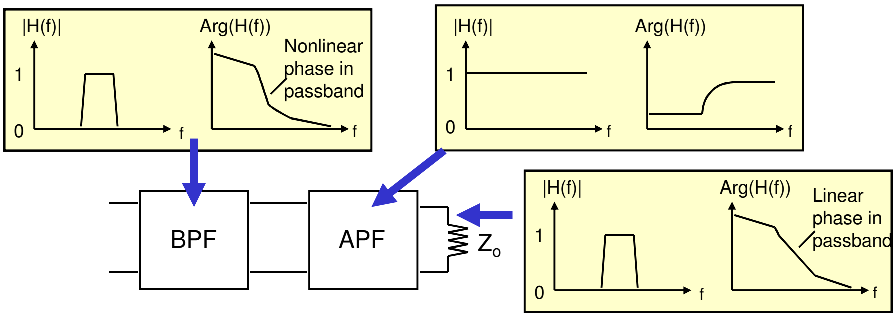


All Pass Filter

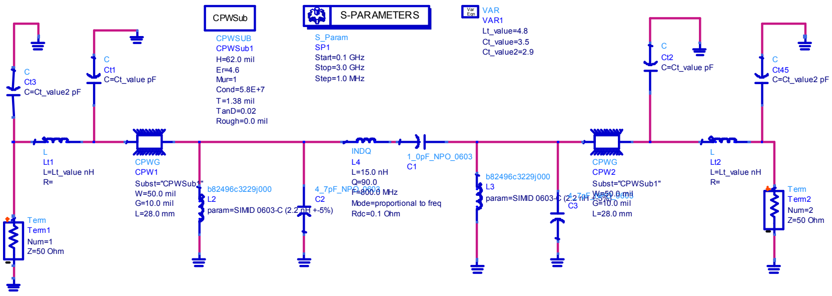
Extra

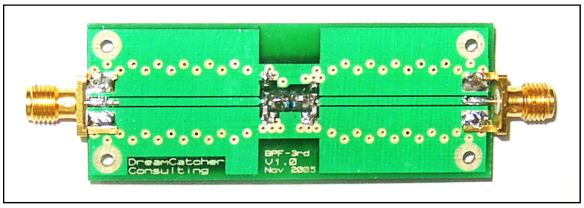
- There is also another class of filter known as All-Pass Filter (APF).
- This type of filter does not produce any attenuation in the magnitude response, but provides phase response in the band of interest.
- APF is often used in conjunction with LPF, BPF, HPF etc to compensate for phase distortion.

Example of APF response



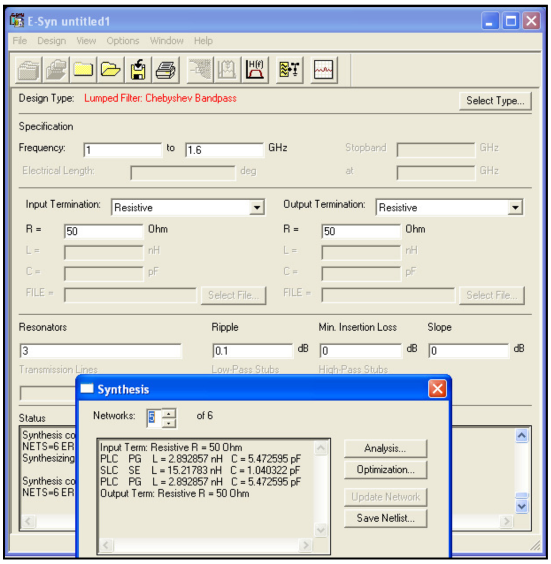
Example 2.4 - Practical RF BPF Design Using SMD Discrete Components



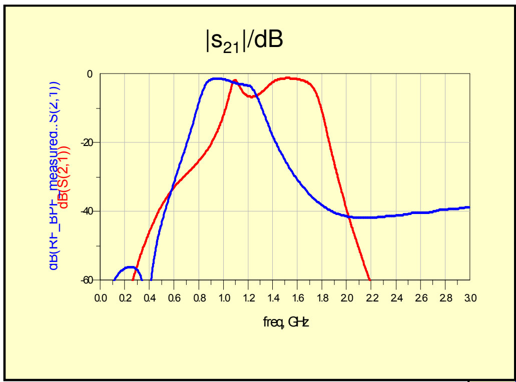


Example 2.4 Cont…

BPF synthesis using synthesis tool E-syn of ADS2003C

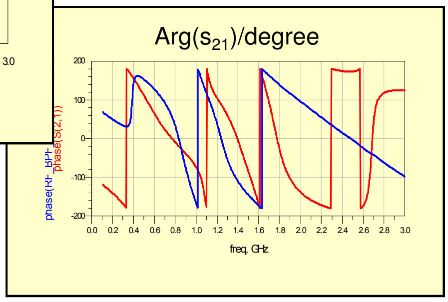


Example 2.4 Cont…



Measured
Simulated

Measurement is performed with Agilent 8753ES Vector Network Analyzer, using Full OSL calibration



3.0 Microwave Filter Realization Using Stripline Structures

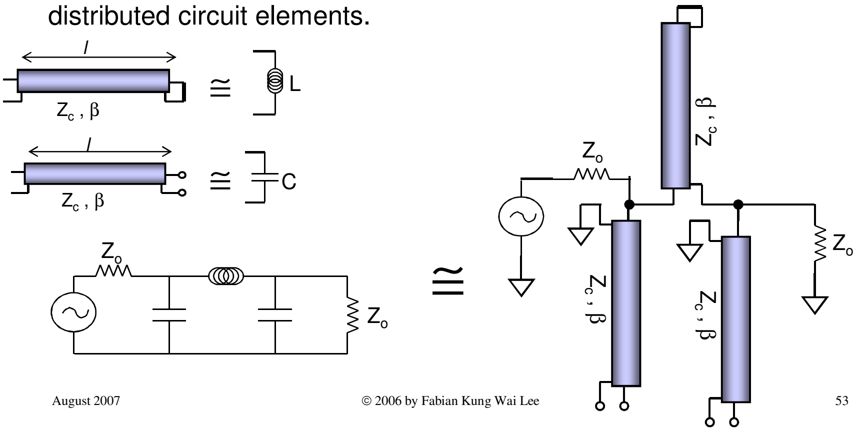
3.1 Basic Approach

Filter Realization Using Distributed Circuit Elements (1)

- Lumped-element filter realization using surface mounted inductors and capacitors generally works well at lower frequency (at UHF, say < 3 GHz).
- At higher frequencies, the practical inductors and capacitors loses their intrinsic characteristics.
- Also a limited range of component values are available from manufacturer.
- Therefore for microwave frequencies (> 3 GHz), passive filter is usually realized using distributed circuit elements such as transmission line sections.
- Here we will focus on stripline microwave circuits.

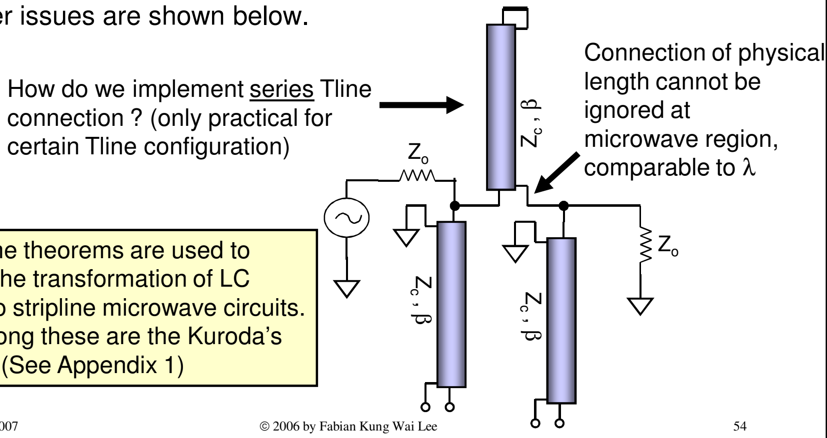
Filter Realization Using Distributed Circuit Elements (2)

- Recall in the study of Terminated Transmission Line Circuit that a length of terminated Tline can be used to approximate an inductor and capacitor.
- This concept forms the basis of transforming the LC passive filter into distributed circuit elements.



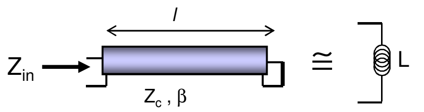
Filter Realization Using Distributed Circuit Elements (3)

- This approach is only approximate. There will be deviation between the actual LC filter response and those implemented with terminated Tline.
- Also the frequency response of distributed circuit filter is periodic.
- Other issues are shown below.



Thus some theorems are used to facilitate the transformation of LC circuit into stripline microwave circuits. Chief among these are the Kuroda's Identities (See Appendix 1)

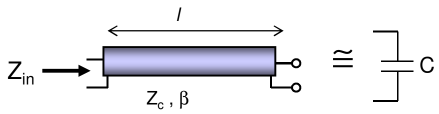
More on Approximating L and C with Terminated Tline: Richard's Transformation



$$Z_{in} = jZ_c \tan(\beta l) = j\omega L = jL\omega$$

$$\tan(\beta l) = \omega \qquad Z_c = L \qquad \textbf{(3.1.1a)}$$

Here instead of fixing Z_c and tuning l to approach an L or C, we allow Z_c to be variable too.



$$Y_{in} = jY_c \tan(\beta l) = j\omega C = jC\omega$$

$$\tan(\beta l) = \omega \qquad Y_c = \frac{1}{Z_c} = C \qquad \textbf{(3.1.1b)}$$

For LPP design, a further requirement is that:

$$\tan(\beta l) = \omega_c = 1$$

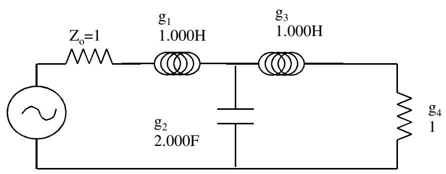
$$\Rightarrow \tan\left(\frac{2\pi}{\lambda_c} l\right) = 1 \Rightarrow l = \frac{\lambda_c}{8} \qquad \textbf{(3.1.1c)}$$

Wavelength at cut-off frequency

Example 3.1 – LPF Design Using Stripline

- Design a 3rd order Butterworth Low-Pass Filter. $R_s = R_L = 50$Ohm, $f_c = 1.5$GHz.

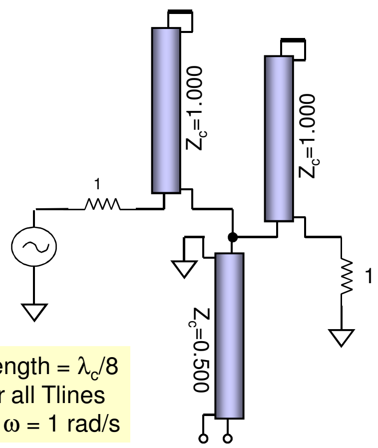
Step 1 & 2: LPP



Step 3: Convert to Tlines

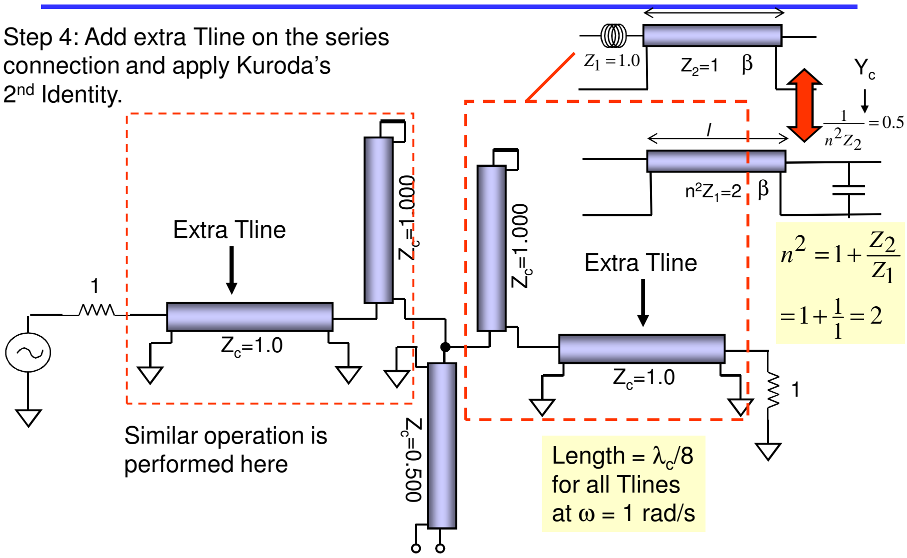
$$\frac{1}{2.000} = 0.500$$

Length = $\lambda_c/8$ for all Tlines at ω = 1 rad/s



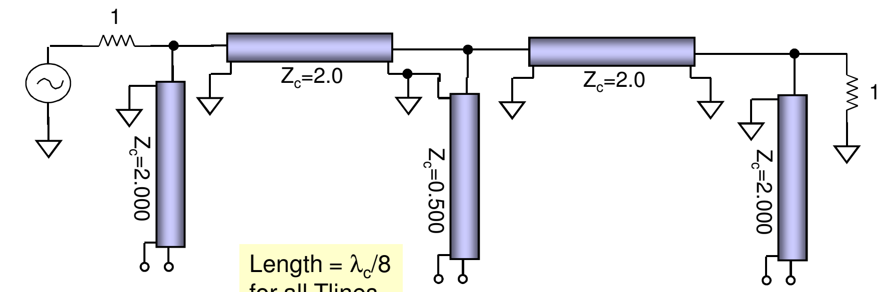
Example 3.1 Cont…

Step 4: Add extra Tline on the series connection and apply Kuroda's 2nd Identity.



Example 3.1 Cont…

After applying Kuroda's 2nd Identity.



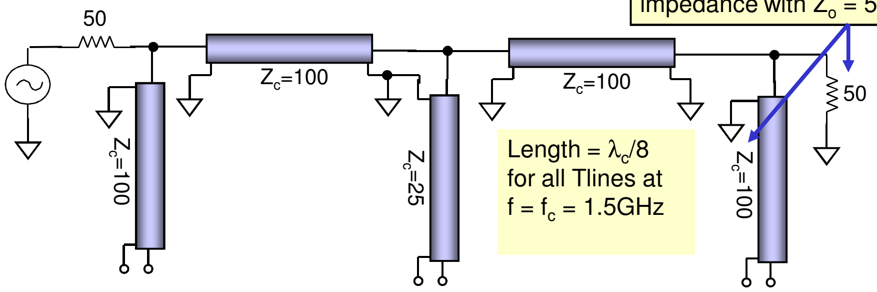
Length = $\lambda_c/8$ for all Tlines at ω = 1 rad/s

Since all Tlines have similar physical length, this approach to stripline filter implementation is also known as Commensurate Line Approach.

Example 3.1 Cont…

Step 5: Impedance and frequency denormalization.

Here we multiply all impedance with Z_o = 50



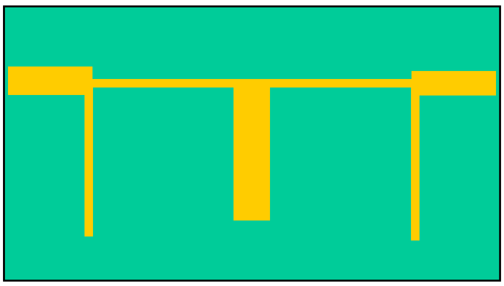
Microstrip line using double-sided FR4 PCB (ε_r = 4.6, H=1.57mm)

Z_c/Ω	$\lambda/8$ @ 1.5GHz /mm	W /mm
50	13.45	2.85
25	12.77	8.00
100	14.23	0.61

We can work out the correct width W given the impedance, dielectric constant and thickness. From W/H ratio, the effective dielectric constant ε_{eff} can be determined. Use this together with frequency at 1.5 GHz to find the wavelength.

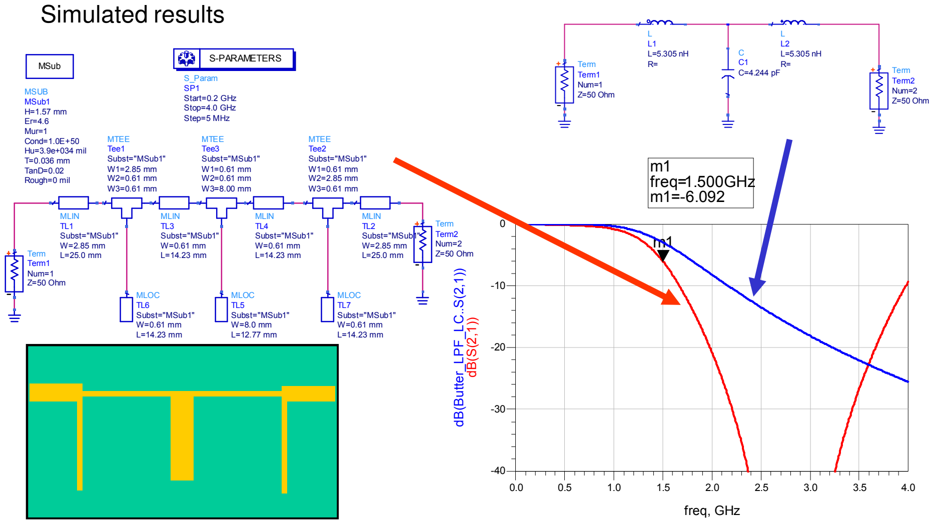
Example 3.1 Cont…

Step 6: The layout (top view)



Example 3.1 Cont…

Simulated results



Conclusions for Section 3.1

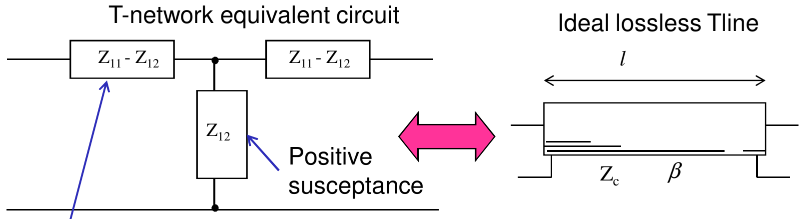
- Further tuning is needed to optimize the frequency response.
- The method just illustrated is good for Low-Pass and Band-Stop filter implementation.
- For High-Pass and Band-Pass, other approaches are needed.

3.2 Further Implementations

Realization of LPF Using Step-Impedance Approach

- A relatively easy way to implement LPF using stripline components.
- Using alternating sections of high and low characteristic impedance tlines to approximate the alternating L and C elements in a LPF.
- Performance of this approach is marginal as it is an approximation, where sharp cutoff is not required.
- As usual beware of parasitic passbands !!!

Equivalent Circuit of a Transmission Line Section



$Z_{11} - Z_{12} = jZ_c\left[\frac{1}{\sin(\beta l)} - \frac{\cos(\beta l)}{\sin(\beta l)}\right]$

$= jZ_c\left[\frac{1-\cos\left(2\left(\frac{\beta l}{2}\right)\right)}{\sin\left(2\left(\frac{\beta l}{2}\right)\right)}\right] = jZ_c\,\frac{2\sin^2\left(2\left(\frac{\beta l}{2}\right)\right)}{2\sin\left(2\left(\frac{\beta l}{2}\right)\right)\cos\left(2\left(\frac{\beta l}{2}\right)\right)}$

$= jZ_c\tan\left(\frac{\beta l}{2}\right)$

Positive reactance

$$Z_{11} = Z_{22} = -jZ_c\cot(\beta l) \quad \textbf{(3.2.1a)}$$

$$Z_{12} = Z_{21} = -jZ_c\operatorname{cosec}(\beta l) \quad \textbf{(3.2.1b)}$$

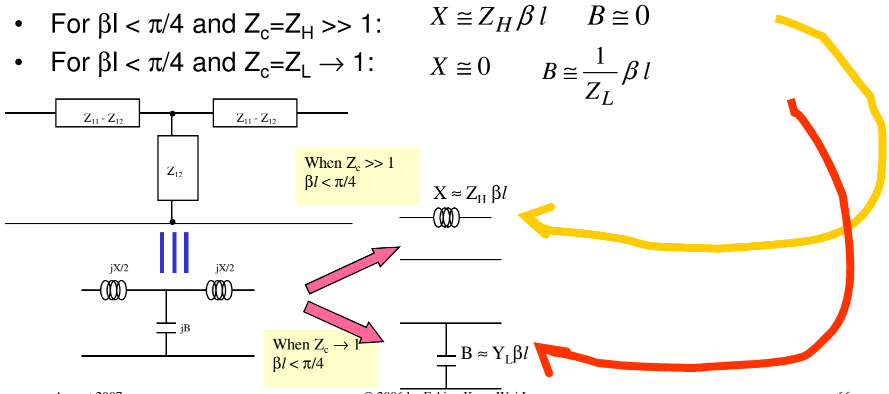
$$\beta \cong \omega\sqrt{\mu_o\varepsilon_e\varepsilon_o} = \sqrt{\varepsilon_e}\,k_o \quad \textbf{(3.2.1c)}$$

Approximation for High and Low Z_C (1)

- When $\beta l < \pi/2$, the series element can be thought of as inductor and the shunt element can be considered a capacitor.

$$|Z_{11} - Z_{12}| = \frac{X}{2} = Z_c\tan\left(\frac{\beta l}{2}\right) \qquad \frac{1}{|Z_{12}|} = B = \frac{1}{Z_c}\sin(\beta l)$$

- For $\beta l < \pi/4$ and $Z_c = Z_H >> 1$: $X \cong Z_H\beta l \quad B \cong 0$
- For $\beta l < \pi/4$ and $Z_c = Z_L \to 1$: $X \cong 0 \quad B \cong \frac{1}{Z_L}\beta l$



Approximation for High and Low Z_C (2)

- Note that $\beta l < \pi/2$ implies a physically short Tline. Thus a short Tline with high Z_c (e.g. Z_H) approximates an inductor.

$$l_L = \frac{\omega_c L}{Z_H \beta} \qquad \textit{(3.2.2a)}$$

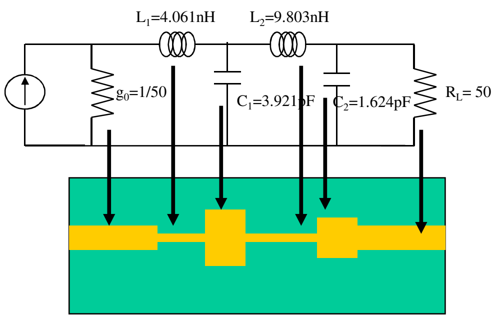
- A short Tline with low Z_c (e.g. Z_L) approximates a capacitor.

$$l_C = \frac{\omega_c C Z_L}{\beta} \qquad \textit{(3.2.2b)}$$

- The ratio of Z_H/Z_L should be as high as possible. Typical values: Z_H = 100 to 150Ω, Z_L = 10 to 15Ω.

Example 3.2 - Mapping LPF Circuit into Step Impedance Tline Network

- For instance consider the LPF Design Example 2.2A (Butterworth).
- Let us use microstrip line. Since a microstrip tline with low Z_c is wide and a tline with high Z_c is narrow, the transformation from circuit to physical layout would be as follows:



Example 3.2 - Physical Realization of LPF

- Using microstrip line, with ε_r = 4.2, d = 1.5mm:

	W/d	d/mm	W/mm	ε_e
$Z_c = 15\Omega$	10.0	1.5	15.0	3.68
$Z_c = 50\Omega$	2.0	1.5	3.0	3.21
$Z_c = 110\Omega$	0.36	1.5	0.6	2.83

$$\beta_L = \sqrt{\varepsilon_{eL}}k_o = \sqrt{\varepsilon_{eL}} \times 2\pi f_c \times 3.3356\times10^{-9} = 60.307 s^{-1}$$

$$\beta_H = \sqrt{\varepsilon_{eH}}k_o = \sqrt{\varepsilon_{eH}} \times 2\pi f_c \times 3.3356\times10^{-9} = 53.258 s^{-1}$$

- L_1=4.061nH, L_2=9.083nH, C_1=3.921pF, C_2=1.624pF.

Example 3.2 - Physical Realization of LPF Cont…

$$l_1 = \frac{\omega_c L_1}{Z_H \beta_H} = 6.5mm$$

$$l_2 = \frac{\omega_c C_1 Z_L}{\beta_L} = 9.2mm$$

$$l_3 = 15.0mm$$

$$l_4 = 3.8mm$$

Verification:

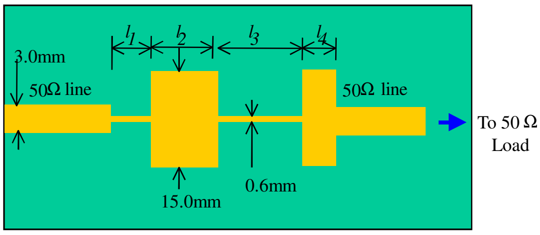
$$\beta_H l_1 = 0.392 < \tfrac{\pi}{4} = 0.7854$$

$$\beta_L l_2 = 0.490 < \tfrac{\pi}{4} = 0.7854$$

$$\beta_H l_3 = 0.905 > \tfrac{\pi}{4} = 0.7854$$

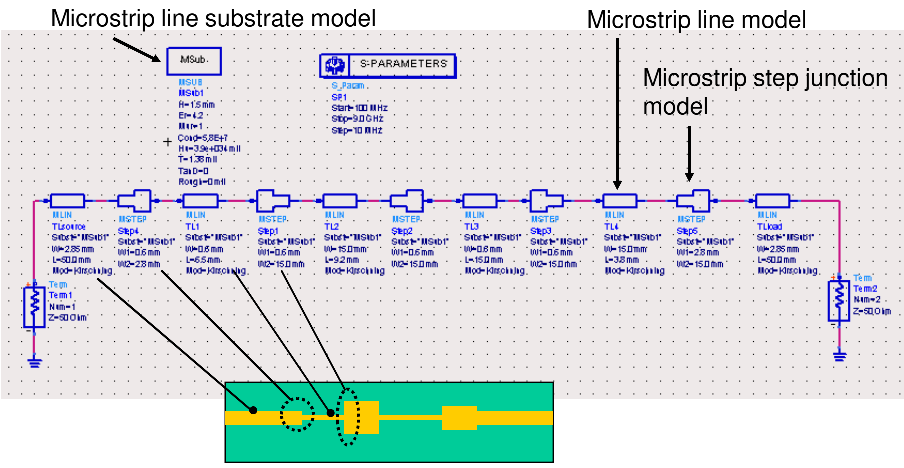
$$\beta_L l_4 = 0.202 < \tfrac{\pi}{4} = 0.7854$$

Nevertheless we still proceed with the imple-mentation. It will be seen that this will affect the accuracy of the -3dB cutoff point of the filter.

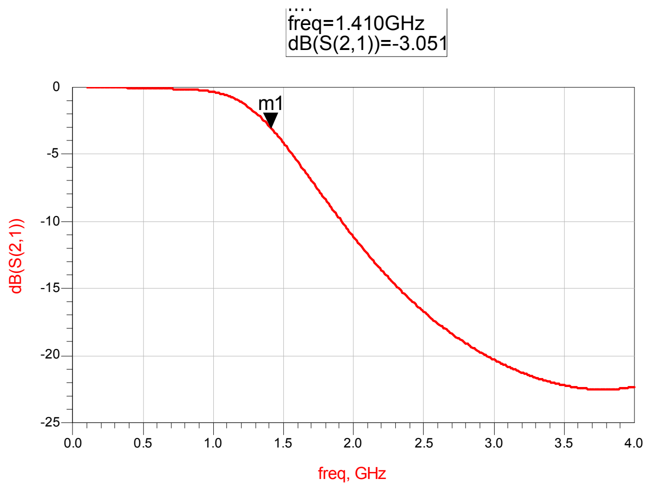


Example 3.2 - Step Impedance LPF Simulation With ADS Software (1)

- Transferring the microstrip line design to ADS:



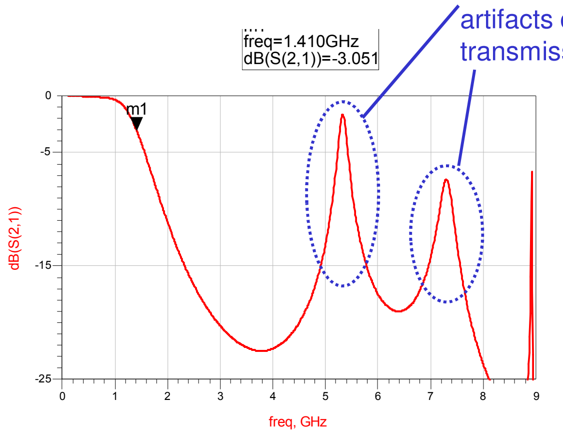
Example 3.2 - Step Impedance LPF Simulation With ADS Software (2)



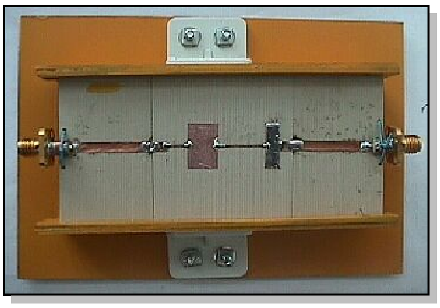
Example 3.2 - Step Impedance LPF Simulation With ADS Software (3)

- However if we extent the stop frequency for the S-parameter simulation to 9GHz...

Parasitic passbands, artifacts due to using transmission lines.

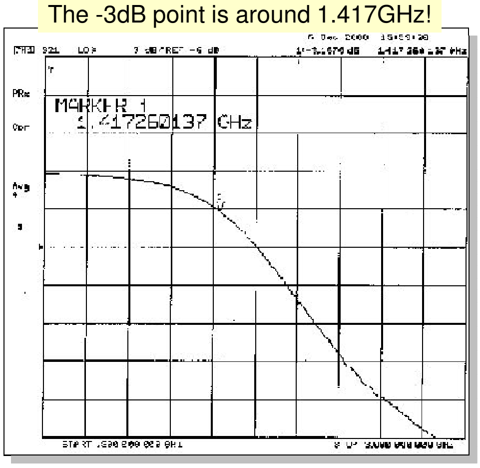


Example 3.2 - Verification with Measurement



The actual LPF constructed in year 2000. Agilent 8720D Vector Network Analyzer is used to perform the S-parameters measurement.

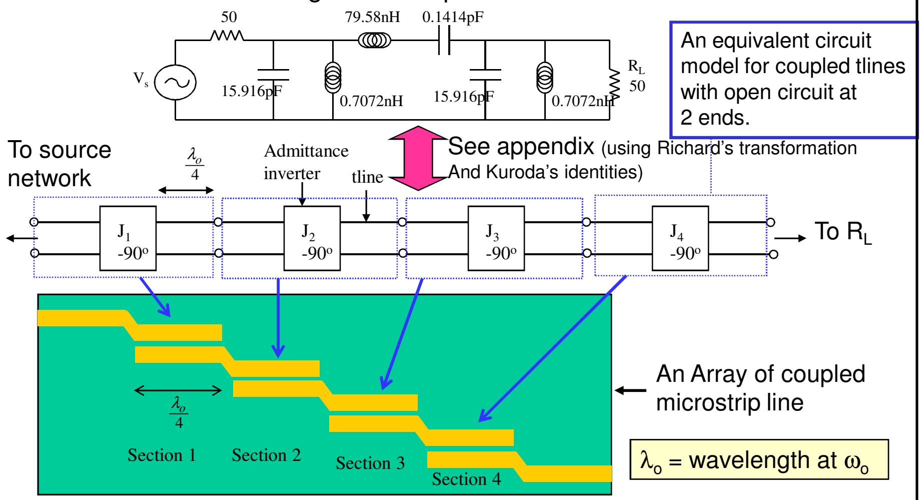
The -3dB point is around 1.417GHz!



Example 3.3 - Realization of BPF Using Coupled StripLine (1)

Extra

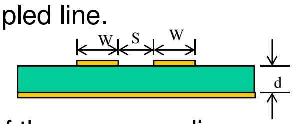
- Based on the BPF design of Example 2.3:



Example 3.3 - Realization of BPF Using Coupled StripLine (2)

Extra

- Each section of the coupled stripline contains three parameters: S, W, d. These parameters can be determined from the values of the odd and even mode impedance (Z_{oo} & Z_{oe}) of each coupled line.



- Z_{oo} and Z_{ee} are in turn depends on the "gain" of the corresponding admittance inverter J.
- And each J_n is given by:

From Example 2.3

$$\omega_1 = 2\pi(1.4GHz)$$
$$\omega_2 = 2\pi(1.6GHz)$$
$$f_o = \sqrt{f_1 f_2} = 1.497GHz$$
$$\Delta = \frac{\omega_2 - \omega_1}{\omega_o} = 0.133$$

$$Z_{oe} = Z_o\left(1 + JZ_o + (JZ_o)^2\right)$$
$$Z_{oo} = Z_o\left(1 - JZ_o + (JZ_o)^2\right)$$

$$J_1 = \frac{1}{Z_o}\sqrt{\frac{\pi\Delta}{2g_1}}$$
$$J_n = \frac{1}{2Z_o}\frac{\pi\Delta}{\sqrt{g_{n-1}g_n}} \quad \text{for} \quad n = 2,3,4\cdots N$$
$$J_{N+1} = \frac{1}{Z_o}\sqrt{\frac{\pi\Delta}{2g_N g_{N+1}}}$$

For derivation see chapter 8, Pozar [2].

Example 3.3 - Realization of BPF Using Coupled StripLine (3)

Extra

Section 1:

$$J_1 = \frac{1}{Z_o}\sqrt{\frac{\pi\Delta}{2g_1}} = 0.009163$$

$$Z_{oe1} = Z_o\left(1 + J_1 Z_o + (J_1 Z_o)^2\right) = 83.403$$
$$Z_{oo1} = Z_o\left(1 - J_1 Z_o + (J_1 Z_o)^2\right) = 37.588$$

Section 2:

$$J_2 = \frac{1}{2Z_o}\frac{\pi\Delta}{\sqrt{g_1 g_2}} = 0.002969$$

$$Z_{oe2} = Z_o\left(1 + J_2 Z_o + (J_2 Z_o)^2\right) = 58.523$$
$$Z_{oo2} = Z_o\left(1 - J_2 Z_o + (J_2 Z_o)^2\right) = 43.680$$

Section 3:

$$J_3 = \frac{1}{2Z_o}\frac{\pi\Delta}{\sqrt{g_2 g_3}} = 0.002969$$

$$Z_{oe3} = 83.403$$
$$Z_{oo3} = 37.588$$

Section 4:

$$J_4 = \frac{1}{Z_o}\sqrt{\frac{\pi\Delta}{2g_3 g_4}} = 0.009163$$

$$Z_{oe4} = 58.523$$
$$Z_{oo4} = 43.680$$

Note:
g_1=1.0000
g_2=2.0000
g_3=1.0000
g_4=1.0000

Example 3.3 - Realization of BPF Using Coupled StripLine (4)

Extra

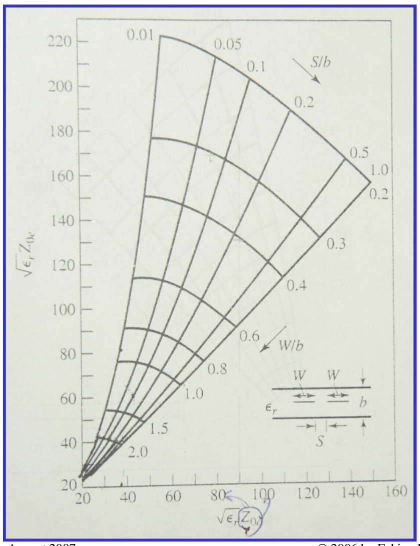
- In this example, edge-coupled stripline is used instead of microstrip line. Stripline does not suffers from dispersion and its propagation mode is pure TEM mode. Hence it is the preferred structured for coupled-line filter.
- From the design data (next slide) for edge-coupled stripline, the parameters W, S and d for each section are obtained.
- Length of each section is *l*.

$$v_p = \frac{1}{\sqrt{\varepsilon_r \varepsilon_o \mu_o}} = 1.463\times 10^8 \qquad \varepsilon_r = 4.2$$

$$l = \frac{v_p}{4f_o} = \frac{1.463\times 10^8}{4\cdot 1.5\times 10^9} = 0.024 \text{ or } 24.0\text{mm}$$

Example 3.3 - Realization of BPF Using Coupled StripLine (5)

Extra



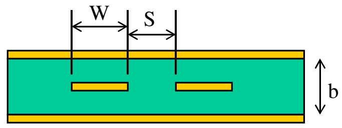
Section 1 and 4:

S/b = 0.07, W/b = 0.3

Section 2 and 3:

S/b = 0.25, W/b = 0.4

By choosing a suitable b, the W and S can be computed.

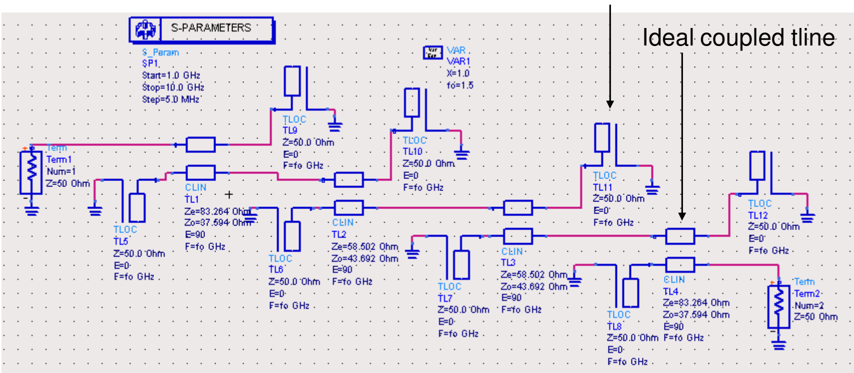


Example 3.3 - Coupled Line BPF Simulation With ADS Software (1)

Extra

- Using ideal transmission line elements:

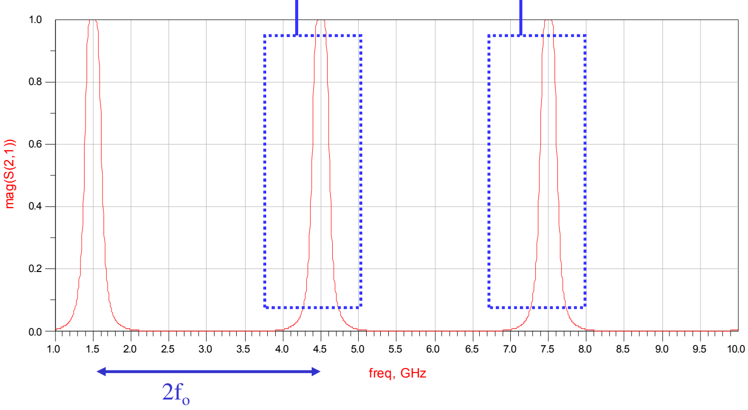
Ideal open circuit



Example 3.3 - Coupled Line BPF Simulation With ADS Software (2)

Extra

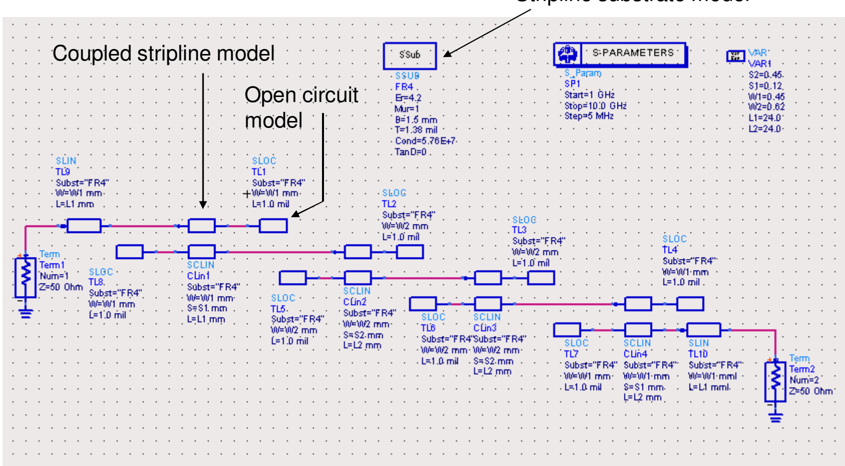
Parasitic passbands. Artifacts due to using distributed elements, these are not present if lumped components are used.



Example 3.3 - Coupled Line BPF Simulation With ADS Software (3)

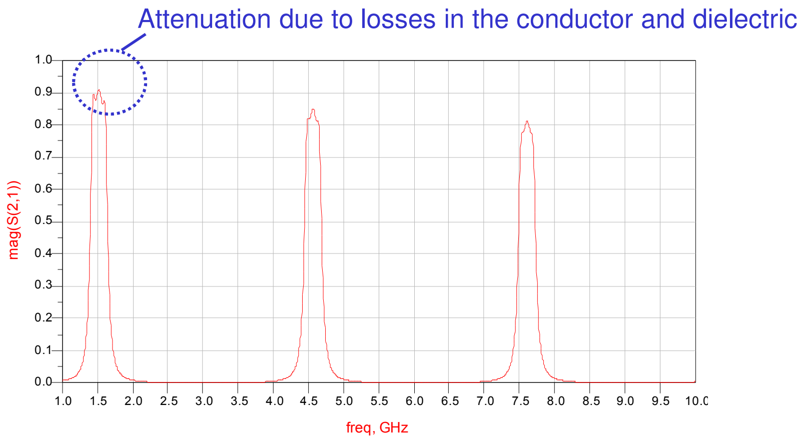
Extra

- Using practical stripline model:



Extra

Example 3.3 - Coupled Line BPF Simulation With ADS Software (4)

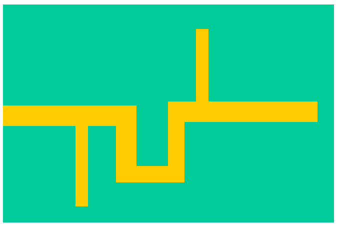


Things You Should Self-Study

- Network analysis and realizability theory ([3] and [6]).
- Synthesis of terminated RLCM one-port circuits ([3] and [6]).
- Ideal impedance and admittance inverters and practical implementation.
- Periodic structures theory ([1] and [2]).
- Filter design by Image Parameter Method (IPM) (Chapter 8, [2]).

Other Types of Stripline Filters (1)

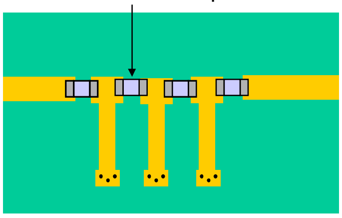
- LPF



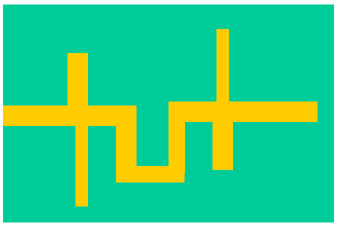
For these delightfully simple approaches see Chapter 43 of [3]

- HPF:

SMD capacitor

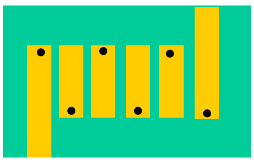


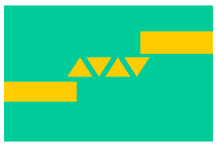
BPF:



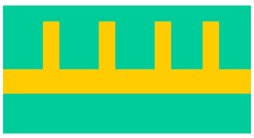
Other Types of Stripline Filters (2)

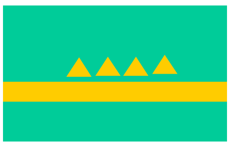
- More BPF:





- BSF:





More information can be obtained from [2], [3], [4] and the book: J. Helszajn, "Microwave planar passive circuits and filters", 1994, John-Wiley & Sons.

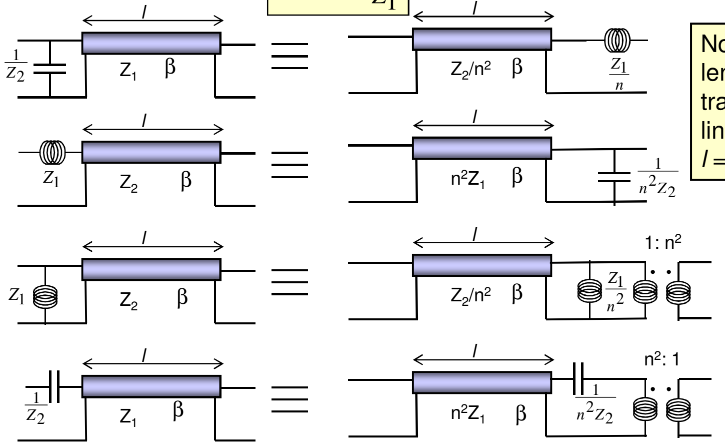
Appendix 1 – Kuroda's Identities

Kuroda's Identities

- As taken from [2].

$$n^2 = 1 + \frac{Z_2}{Z_1}$$

Note: The inductor represents shorted Tline while the capacitor represents open-circuit Tline.



Note: the length of all transmission lines is $l = \lambda/8$

THE END