Chapter 6

Optical Amplifiers

As seen in Chapter 5, the transmission distance of any fiber-optic communication system is eventually limited by fiber losses. For long-haul systems, the loss limitation has traditionally been overcome using optoelectronic repeaters in which the optical signal is first converted into an electric current and then regenerated using a transmitter. Such regenerators become quite complex and expensive for wavelength-division multiplexed (WDM) lightwave systems. An alternative approach to loss management makes use of optical amplifiers, which amplify the optical signal directly without requiring its conversion to the electric domain. Several kinds of optical amplifiers were developed during the 1980s, and the use of optical amplifiers for long-haul lightwave systems became widespread during the 1990s. By 1996, optical amplifiers were a part of the fiber-optic cables laid across the Atlantic and Pacific oceans. This chapter is devoted to optical amplifiers. In Section 6.1 we discuss general concepts common to all optical amplifiers. Semiconductor optical amplifiers are considered in Section 6.2, while Section 6.3 focuses on Raman amplifiers. Section 6.4 is devoted to fiber amplifiers made by doping the fiber core with a rare-earth element. The emphasis is on the erbium-doped fiber amplifiers, used almost exclusively for 1.55-µm lightwave systems. System applications of optical amplifiers are discussed in Section 6.5.

6.1 Basic Concepts

Most optical amplifiers amplify incident light through stimulated emission, the same mechanism that is used by lasers (see Section 3.1). Indeed, an optical amplifier is nothing but a laser without feedback. Its main ingredient is the optical gain realized when the amplifier is pumped (optically or electrically) to achieve population inversion. The optical gain, in general, depends not only on the frequency (or wavelength) of the incident signal, but also on the local beam intensity at any point inside the amplifier. Details of the frequency and intensity dependence of the optical gain depend on the amplifier medium. To illustrate the general concepts, let us consider the case in which the gain medium is modeled as a homogeneously broadened two-level system. The
gain coefficient of such a medium can be written as [1]

\[
g(\omega) = \frac{g_0}{1 + \left(\omega - \omega_0\right)^2 T_2^2 + P/P_s},
\]

(6.1.1)

where \(g_0\) is the peak value of the gain, \(\omega\) is the optical frequency of the incident signal, \(\omega_0\) is the atomic transition frequency, and \(P\) is the optical power of the signal being amplified. The saturation power \(P_s\) depends on gain-medium parameters such as the fluorescence time \(T_1\) and the transition cross section; its expression for different kinds of amplifiers is given in the following sections. The parameter \(T_2\) in Eq. (6.1.1), known as the dipole relaxation time, is typically quite small (\(< 1\) ps). The fluorescence time \(T_1\), also called the population relaxation time, varies in the range 100 ps–10 ms, depending on the gain medium. Equation (6.1.1) can be used to discuss important characteristics of optical amplifiers, such as the gain bandwidth, amplification factor, and output saturation power.

### 6.1.1 Gain Spectrum and Bandwidth

Consider the unsaturated regime in which \(P/P_s \ll 1\) throughout the amplifier. By neglecting the term \(P/P_s\) in Eq. (6.1.1), the gain coefficient becomes

\[
g(\omega) = \frac{g_0}{1 + \left(\omega - \omega_0\right)^2 T_2^2}.
\]

(6.1.2)

This equation shows that the gain is maximum when the incident frequency \(\omega\) coincides with the atomic transition frequency \(\omega_0\). The gain reduction for \(\omega \neq \omega_0\) is governed by a Lorentzian profile that is a characteristic of homogeneously broadened two-level systems [1]. As discussed later, the gain spectrum of actual amplifiers can deviate considerably from the Lorentzian profile. The gain bandwidth is defined as the full width at half maximum (FWHM) of the gain spectrum \(g(\omega)\). For the Lorentzian spectrum, the gain bandwidth is given by \(\Delta \nu_g = 2/\pi T_2\), or by

\[
\Delta \nu_g = \frac{\Delta \omega_g}{2\pi} = \frac{1}{\pi T_2}.
\]

(6.1.3)

As an example, \(\Delta \nu_g \sim 5\) THz for semiconductor optical amplifiers for which \(T_2 \sim 60\) fs. Amplifiers with a relatively large bandwidth are preferred for optical communication systems because the gain is then nearly constant over the entire bandwidth of even a multichannel signal.

The concept of amplifier bandwidth is commonly used in place of the gain bandwidth. The difference becomes clear when one considers the amplifier gain \(G\), known as the amplification factor and defined as

\[
G = P_{\text{out}}/P_{\text{in}}.
\]

(6.1.4)

where \(P_{\text{in}}\) and \(P_{\text{out}}\) are the input and output powers of the continuous-wave (CW) signal being amplified. We can obtain an expression for \(G\) by using

\[
\frac{dP}{dz} = gP,
\]

(6.1.5)
where $P(z)$ is the optical power at a distance $z$ from the input end. A straightforward integration with the initial condition $P(0) = P_{in}$ shows that the signal power grows exponentially as

$$P(z) = P_{in} \exp(gz). \quad (6.1.6)$$

By noting that $P(L) = P_{out}$ and using Eq. (6.1.4), the amplification factor for an amplifier of length $L$ is given by

$$G(\omega) = \exp[g(\omega)L], \quad (6.1.7)$$

where the frequency dependence of both $G$ and $g$ is shown explicitly. Both the amplifier gain $G(\omega)$ and the gain coefficient $g(\omega)$ are maximum when $\omega = \omega_0$ and decrease with the signal detuning $\omega - \omega_0$. However, $G(\omega)$ decreases much faster than $g(\omega)$. The amplifier bandwidth $\Delta V_A$ is defined as the FWHM of $G(\omega)$ and is related to the gain bandwidth $\Delta V_g$ as

$$\Delta V_A = \Delta V_g \left[ \frac{\ln 2}{\ln(G_0/2)} \right]^{1/2}, \quad (6.1.8)$$

where $G_0 = \exp(g_0L)$. Figure 6.1 shows the gain profile $g(\omega)$ and the amplification factor $G(\omega)$ by plotting $g/g_0$ and $G/G_0$ as a function of $(\omega - \omega_0)/T_2$. The amplifier bandwidth is smaller than the gain bandwidth, and the difference depends on the amplifier gain itself.
6.1. Gain Saturation

The origin of gain saturation lies in the power dependence of the $g(\omega)$ in Eq. (6.1.1). Since $g$ is reduced when $P$ becomes comparable to $P_s$, the amplification factor $G$ decreases with an increase in the signal power. This phenomenon is called gain saturation. Consider the case in which incident signal frequency is exactly tuned to the gain peak ($\omega = \omega_0$). The detuning effects can be incorporated in a straightforward manner. By substituting $g$ from Eq. (6.1.1) in Eq. (6.1.5), we obtain

$$
\frac{dP}{dz} = g_0 P + \frac{P}{P_s}.
$$

(6.1.9)

This equation can easily be integrated over the amplifier length. By using the initial condition $P(0) = P_{in}$ together with $P(L) = P_{out} = G P_{in}$, we obtain the following implicit relation for the large-signal amplifier gain:

$$
G = G_0 \exp \left( - \frac{G - 1}{G_0} \frac{P_{out}}{P_s} \right).
$$

(6.1.10)

Equation (6.1.10) shows that the amplification factor $G$ decreases from its unsaturated value $G_0$ when $P_{out}$ becomes comparable to $P_s$. Figure 6.2 shows the saturation characteristics by plotting $G$ as a function of $P_{out}/P_s$ for several values of $G_0$. A quantity of practical interest is the output saturation power $P_{out}^*$, defined as the output power for which the amplifier gain $G$ is reduced by a factor of 2 (or by 3 dB) from its unsaturated value $G_0$. By using $G = G_0/2$ in Eq. (6.1.10),

$$
P_{out}^* = \frac{G_0 \ln 2}{G_0 - 2} P_s.
$$

(6.1.11)
Here, $P_{\text{out}}^\text{opt}$ is smaller than $P_t$ by about 30%. Indeed, by noting that $G_0 \gg 2$ in practice ($G_0 = 1000$ for 30-dB amplifier gain), $P_{\text{out}}^\text{opt} \approx (\ln 2)P_t \approx 0.69P_t$. As seen in Fig. 6.2, $P_{\text{out}}^\text{opt}$ becomes nearly independent of $G_0$ for $G_0 > 20$ dB.

### 6.1.3 Amplifier Noise

All amplifiers degrade the signal-to-noise ratio (SNR) of the amplified signal because of spontaneous emission that adds noise to the signal during its amplification. The SNR degradation is quantified through a parameter $F_n$, called the amplifier noise figure in analogy with the electronic amplifiers (see Section 4.4.1) and defined as [2]

$$F_n = \frac{(\text{SNR})_\text{in}}{(\text{SNR})_\text{out}}, \quad (6.1.12)$$

where SNR refers to the electric power generated when the optical signal is converted into an electric current. In general, $F_n$ depends on several detector parameters that govern thermal noise associated with the detector (see Section 4.4.1). A simple expression for $F_n$ can be obtained by considering an ideal detector whose performance is limited by shot noise only [2].

Consider an amplifier with the gain $G$ such that the output and input powers are related by $P_{\text{out}} = GP_{\text{in}}$. The SNR of the input signal is given by

$$(\text{SNR})_\text{in} = \frac{(I)^2}{\sigma_i^2} = \frac{(RP_{\text{in}})^2}{2q(RP_{\text{in}})\Delta f} = \frac{P_{\text{in}}}{2h\nu\Delta f}, \quad (6.1.13)$$

where $\langle I \rangle = RP_{\text{in}}$ is the average photocurrent, $R = q/h\nu$ is the responsivity of an ideal photodetector with unit quantum efficiency (see Section 4.1), and

$$\sigma_i^2 = 2q(RP_{\text{in}})\Delta f \quad (6.1.14)$$

is obtained from Eq. (4.4.5) for the shot noise by setting the dark current $I_d = 0$. Here $\Delta f$ is the detector bandwidth. To evaluate the SNR of the amplified signal, one should add the contribution of spontaneous emission to the receiver noise.

The spectral density of spontaneous-emission-induced noise is nearly constant (white noise) and can be written as [2]

$$S_{sp}(\nu) = (G - 1)n_{sp}\nu, \quad (6.1.15)$$

where $\nu$ is the optical frequency. The parameter $n_{sp}$ is called the spontaneous-emission factor (or the population-inversion factor) and is given by

$$n_{sp} = N_2/(N_2 - N_1), \quad (6.1.16)$$

where $N_1$ and $N_2$ are the atomic populations for the ground and excited states, respectively. The effect of spontaneous emission is to add fluctuations to the amplified signal; these are converted to current fluctuations during the photodetection process.

It turns out that the dominant contribution to the receiver noise comes from the beating of spontaneous emission with the signal [2]. The spontaneously emitted radiation
mixes with the amplified signal and produces the current \( I = R |\sqrt{GE_{in}} + E_{sp}^2 |^2 \) at the photodetector of responsivity \( R \). Noting that \( E_{in} \) and \( E_{sp} \) oscillate at different frequencies with a random phase difference, it is easy to see that the beating of spontaneous emission with the signal will produce a noise current \( \Delta I = 2R(GP_{in})^{1/2}|E_{sp}|\cos \theta \), where \( \theta \) is a rapidly varying random phase. Averaging over the phase, and neglecting all other noise sources, the variance of the photocurrent can be written as

\[
\sigma^2 \approx 4(RGP_{in})(RS_{sp})\Delta f, \tag{6.1.17}
\]

where \( \cos^2 \theta \) was replaced by its average value \( \frac{1}{2} \). The SNR of the amplified signal is thus given by

\[
(SNR)_{out} = \frac{(I)^2}{\sigma^2} = \frac{(RGP_{in})^2}{\sigma^2} \approx \frac{GP_{in}}{4S_{sp}\Delta f}. \tag{6.1.18}
\]

The amplifier noise figure can now be obtained by substituting Eqs. (6.1.13) and (6.1.18) in Eq. (6.1.12). If we also use Eq. (6.1.15) for \( S_{sp} \),

\[
F_n = 2n_{sp}(G - 1)/G \approx 2n_{sp}. \tag{6.1.19}
\]

This equation shows that the SNR of the amplified signal is degraded by 3 dB even for an ideal amplifier for which \( n_{sp} = 1 \). For most practical amplifiers, \( F_n \) exceeds 3 dB and can be as large as 6–8 dB. For its application in optical communication systems, an optical amplifier should have \( F_n \) as low as possible.

### 6.1.4 Amplifier Applications

Optical amplifiers can serve several purposes in the design of fiber-optic communication systems: three common applications are shown schematically in Fig. 6.3. The most important application for long-haul systems consists of using amplifiers as in-line amplifiers which replace electronic regenerators (see Section 5.1). Many optical amplifiers can be cascaded in the form of a periodic chain as long as the system performance is not limited by the cumulative effects of fiber dispersion, fiber nonlinearity, and amplifier noise. The use of optical amplifiers is particularly attractive for WDM lightwave systems as all channels can be amplified simultaneously.

Another way to use optical amplifiers is to increase the transmitter power by placing an amplifier just after the transmitter. Such amplifiers are called power amplifiers or power boosters, as their main purpose is to boost the power transmitted. A power amplifier can increase the transmission distance by 100 km or more depending on the amplifier gain and fiber losses. Transmission distance can also be increased by putting an amplifier just before the receiver to boost the received power. Such amplifiers are called optical preamplifiers and are commonly used to improve the receiver sensitivity. Another application of optical amplifiers is to use them for compensating distribution losses in local-area networks. As discussed in Section 5.1, distribution losses often limit the number of nodes in a network. Many other applications of optical amplifiers are discussed in Chapter 8 devoted to WDM lightwave systems.
Figure 6.3: Three possible applications of optical amplifiers in lightwave systems: (a) as in-line amplifiers; (b) as a booster of transmitter power; (c) as a preamplifier to the receiver.

6.2 Semiconductor Optical Amplifiers

All lasers act as amplifiers close to but before reaching threshold, and semiconductor lasers are no exception. Indeed, research on semiconductor optical amplifiers (SOAs) started soon after the invention of semiconductor lasers in 1962. However, it was only during the 1980s that SOAs were developed for practical applications, motivated largely by their potential applications in lightwave systems [3]–[8]. In this section we discuss the amplification characteristics of SOAs and their applications.

6.2.1 Amplifier Design

The amplifier characteristics discussed in Section 6.1 were for an optical amplifier without feedback. Such amplifiers are called traveling-wave (TW) amplifiers to emphasize that the amplified signal travels in the forward direction only. Semiconductor lasers experience a relatively large feedback because of reflections occurring at the cleaved facets (32% reflectivity). They can be used as amplifiers when biased below threshold, but multiple reflections at the facets must be included by considering a Fabry–Perot (FP) cavity. Such amplifiers are called FP amplifiers. The amplification factor is obtained by using the standard theory of FP interferometers and is given by [4]

$$G_{FP}(v) = \frac{(1 - R_1)(1 - R_2)G(v)}{(1 - G\sqrt{R_1R_2})^2 + 4G\sqrt{R_1R_2}\sin^2[\pi(v - v_m)/\Delta v_L]}.$$  (6.2.1)
where $R_1$ and $R_2$ are the facet reflectivities, $v_m$ represents the cavity-resonance frequencies [see Eq. (3.3.5)], and $\Delta v_L$ is the longitudinal-mode spacing, also known as the free spectral range of the FP cavity. The single-pass amplification factor $G$ corresponds to that of a TW amplifier and is given by Eq. (6.1.7) when gain saturation is negligible. Indeed, $G_{FP}$ reduces to $G$ when $R_1 = R_2 = 0$.

As evident from Eq. (6.2.1), $G_{FP}(v)$ peaks whenever $v$ coincides with one of the cavity-resonance frequencies and drops sharply in between them. The amplifier bandwidth is thus determined by the sharpness of the cavity resonance. One can calculate the amplifier bandwidth from the detuning $v - v_m$ for which $G_{FP}$ drops by 3 dB from its peak value. The result is given by

$$\Delta v_A = \frac{2 \Delta v_m}{\pi} \sin^{-1} \left( \frac{1 - G \sqrt{R_1 R_2}}{(4G \sqrt{R_1 R_2})^{1/2}} \right).$$

(6.2.2)

To achieve a large amplification factor, $G \sqrt{R_1 R_2}$ should be quite close to 1. As seen from Eq. (6.2.2), the amplifier bandwidth is then a small fraction of the free spectral range of the FP cavity (typically, $\Delta v_L \sim 100$ GHz and $\Delta v_A < 10$ GHz). Such a small bandwidth makes FP amplifiers unsuitable for most lightwave system applications.

TW-type SOAs can be made if the reflection feedback from the end facets is suppressed. A simple way to reduce the reflectivity is to coat the facets with an antireflection coating. However, it turns out that the reflectivity must be extremely small (<0.1%) for the SOA to act as a TW amplifier. Furthermore, the minimum reflectivity depends on the amplifier gain itself. One can estimate the tolerable value of the facet reflectivity by considering the maximum and minimum values of $G_{FP}$ from Eq. (6.2.1) near a cavity resonance. It is easy to verify that their ratio is given by

$$\Delta G = \frac{G_{FP}^{\text{max}}}{G_{FP}^{\text{min}}} = \left( \frac{1 + G \sqrt{R_1 R_2}}{1 - G \sqrt{R_1 R_2}} \right)^2.$$ (6.2.3)

If $\Delta G$ exceeds 3 dB, the amplifier bandwidth is set by the cavity resonances rather than by the gain spectrum. To keep $\Delta G < 2$, the facet reflectivities should satisfy the condition

$$G \sqrt{R_1 R_2} < 0.17.$$ (6.2.4)

It is customary to characterize the SOA as a TW amplifier when Eq. (6.2.4) is satisfied. A SOA designed to provide a 30-dB amplification factor ($G = 1000$) should have facet reflectivities such that $\sqrt{R_1 R_2} < 1.7 \times 10^{-4}$.

Considerable effort is required to produce antireflection coatings with reflectivities less than 0.1%. Even then, it is difficult to obtain low facet reflectivities in a predictable and regular manner. For this reason, alternative techniques have been developed to reduce the reflection feedback in SOAs. In one method, the active-region stripe is tilted from the facet normal, as shown in Fig. 6.4(a). Such a structure is referred to as the angled-facet or tilted-stripe structure [9]. The reflected beam at the facet is physically separated from the forward beam because of the angled facet. Some feedback can still occur, as the optical mode spreads beyond the active region in all semiconductor laser devices. In practice, the combination of an antireflection coating and the tilted stripe can produce reflectivities below $10^{-3}$ (as small as $10^{-4}$ with design optimization). In
an alternative scheme [10] a transparent region is inserted between the active-layer ends and the facets [see Fig. 6.4(b)]. The optical beam spreads in this window region before arriving at the semiconductor–air interface. The reflected beam spreads even further on the return trip and does not couple much light into the thin active layer. Such a structure is called buried-facet or window-facet structure and has provided reflectivities as small as $10^{-4}$ when used in combination with antireflection coatings.

### 6.2.2 Amplifier Characteristics

The amplification factor of SOAs is given by Eq. (6.2.1). Its frequency dependence results mainly from the frequency dependence of $G(\nu)$ when condition (6.2.4) is satisfied. The measured amplifier gain exhibits ripples reflecting the effects of residual facet reflectivities. Figure 6.5 shows the wavelength dependence of the amplifier gain measured for a SOA with the facet reflectivities of about $4 \times 10^{-4}$. Condition (6.2.4) is well satisfied as $G\sqrt{R_1 R_2} \approx 0.04$ for this amplifier. Gain ripples were negligibly small as the SOA operated in a nearly TW mode. The 3-dB amplifier bandwidth is about 70 nm because of a relatively broad gain spectrum of SOAs (see Section 3.3.1).

To discuss gain saturation, consider the peak gain and assume that it increases linearly with the carrier population $N$ as (see Section 3.3.1)

$$g(N) = (\Gamma \sigma_g / V)(N - N_0),$$

(6.2.5)
where $\Gamma$ is the confinement factor, $\sigma_g$ is the differential gain, $V$ is the active volume, and $N_0$ is the value of $N$ required at transparency. The gain has been reduced by $\Gamma$ to account for spreading of the waveguide mode outside the gain region of SOAs. The carrier population $N$ changes with the injection current $I$ and the signal power $P$ as indicated in Eq. (3.5.2). Expressing the photon number in terms of the optical power, this equation can be written as

$$\frac{dN}{dt} = I - N \frac{\tau_c}{\gamma} - \sigma_g \sigma_m h v P,$$

where $\tau_c$ is the carrier lifetime and $\sigma_m$ is the cross-sectional area of the waveguide mode. In the case of a CW beam, or pulses much longer than $\tau_c$, the steady-state value of $N$ can be obtained by setting $dN/dt = 0$ in Eq. (6.2.6). When the solution is substituted in Eq. (6.2.5), the optical gain is found to saturate as

$$g = \frac{g_0}{1 + P/P_s},$$

where the small-signal gain $g_0$ is given by

$$g_0 = (\Gamma \sigma_g/V)(I \tau_c/q - N_0),$$

and the saturation power $P_s$ is defined as

$$P_s = h v \sigma_m / (\Gamma \tau_c).$$

A comparison of Eqs. (6.1.1) and (6.2.7) shows that the SOA gain saturates in the same way as that for a two-level system. Thus, the output saturation power $P_{out}$ is obtained.
from Eq. (6.1.11) with \( P_s \) given by Eq. (6.2.9). Typical values of \( P_{\text{out}} \) are in the range 5–10 mW.

The noise figure \( F_n \) of SOAs is larger than the minimum value of 3 dB for several reasons. The dominant contribution comes from the spontaneous-emission factor \( n_{sp} \). For SOAs, \( n_{sp} \) is obtained from Eq. (6.1.16) by replacing \( N_2 \) and \( N_1 \) by \( N \) and \( N_0 \), respectively. An additional contribution results from internal losses (such as free-carrier absorption or scattering loss) which reduce the available gain from \( g \) to \( g - \alpha_{\text{int}} \). By using Eq. (6.1.19) and including this additional contribution, the noise figure can be written as \[ F_n = 2 \left( \frac{N}{N - N_0} \right) \left( \frac{g}{g - \alpha_{\text{int}}} \right). \] (6.2.10)

Residual facet reflectivities increase \( F_n \) by an additional factor that can be approximated by \( 1 + R_1 G \), where \( R_1 \) is the reflectivity of the input facet [6]. In most TW amplifiers, \( R_1 G \ll 1 \), and this contribution can be neglected. Typical values of \( F_n \) for SOAs are in the range 5–7 dB.

An undesirable characteristic of SOAs is their polarization sensitivity. The amplifier gain \( G \) differs for the transverse electric and magnetic (TE, TM) modes by as much as 5–8 dB simply because both \( G \) and \( \sigma_g \) are different for the two orthogonally polarized modes. This feature makes the amplifier gain sensitive to the polarization state of the input beam, a property undesirable for lightwave systems in which the state of polarization changes with propagation along the fiber (unless polarization-maintaining fibers are used). Several schemes have been devised to reduce the polarization sensitivity [10]–[15]. In one scheme, the amplifier is designed such that the width and the thickness of the active region are comparable. A gain difference of less than 1.3 dB between TE and TM polarizations has been realized by making the active layer 0.26 \( \mu \)m thick and 0.4 \( \mu \)m wide [10]. Another scheme makes use of a large-optical-cavity structure; a gain difference of less than 1 dB has been obtained with such a structure [11].

Several other schemes reduce the polarization sensitivity by using two amplifiers or two passes through the same amplifier. Figure 6.6 shows three such configurations. In Fig. 6.6(a), the TE-polarized signal in one amplifier becomes TM polarized in the second amplifier, and vice versa. If both amplifiers have identical gain characteristics, the twin-amplifier configuration provides signal gain that is independent of the signal polarization. A drawback of the series configuration is that residual facet reflectivities lead to mutual coupling between the two amplifiers. In the parallel configuration shown in Fig. 6.6(b) the incident signal is split into a TE- and a TM-polarized signal, each of which is amplified by separate amplifiers. The amplified TE and TM signals are then combined to produce the amplified signal with the same polarization as that of the input beam [12]. The double-pass configuration of Fig. 6.6(c) passes the signal through the same amplifier twice, but the polarization is rotated by 90° between the two passes [13]. Since the amplified signal propagates in the backward direction, a 3-dB fiber coupler is needed to separate it from the incident signal. Despite a 6-dB loss occurring at the fiber coupler (3 dB for the input signal and 3 dB for the amplified signal) this configuration provides high gain from a single amplifier, as the same amplifier supplies gain on the two passes.
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Figure 6.6: Three configurations used to reduce the polarization sensitivity of semiconductor optical amplifiers: (a) twin amplifiers in series; (b) twin amplifiers in parallel; and (c) double pass through a single amplifier.

6.2.3 Pulse Amplification

One can adapt the formulation developed in Section 2.4 for pulse propagation in optical fibers to the case of SOAs by making a few changes. The dispersive effects are not important for SOAs because of negligible material dispersion and a short amplifier length (<1 mm in most cases). The amplifier gain can be included by adding the term $gA/2$ on the right side of Eq. (2.4.7). By setting $\beta_2 = \beta_3 = 0$, the amplitude $A(z,t)$ of the pulse envelope then evolves as [18]

$$\frac{\partial A}{\partial z} + \frac{1}{\nu_g} \frac{\partial A}{\partial t} = \frac{1}{2} (1 - i\beta_c)gA,$$

(6.2.11)

where carrier-induced index changes are included through the linewidth enhancement factor $\beta_c$ (see Section 3.5.2). The time dependence of $g$ is governed by Eqs. (6.2.5) and (6.2.6). The two equations can be combined to yield

$$\frac{\partial g}{\partial t} = \frac{g_0 - g}{\tau_g} - \frac{g|A|^2}{E_{\text{sat}}},$$

(6.2.12)

where the saturation energy $E_{\text{sat}}$ is defined as

$$E_{\text{sat}} = \hbar \nu (\sigma_m / \sigma_g),$$

(6.2.13)

and $g_0$ is given by Eq. (6.2.8). Typically $E_{\text{sat}} \sim 1 \text{ pJ}$. 

Equations (6.2.11) and (6.2.12) govern amplification of optical pulses in SOAs. They can be solved analytically for pulses whose duration is short compared with the carrier lifetime \( \tau_p \ll \tau_c \). The first term on the right side of Eq. (6.2.12) can then be neglected during pulse amplification. By introducing the reduced time \( \tau = t - z/v_g \) together with \( A = \sqrt{\mathcal{P}} \exp(i\phi) \), Eqs. (6.2.11) and (6.2.12) can be written as [18]

\[
\frac{\partial P}{\partial z} = g(z, \tau)P(z, \tau), \tag{6.2.14}
\]

\[
\frac{\partial \phi}{\partial z} = -\frac{1}{2} \beta_c g(z, \tau), \tag{6.2.15}
\]

\[
\frac{\partial g}{\partial \tau} = -g(z, \tau)P(z, \tau)/E_{\text{sat}}. \tag{6.2.16}
\]

Equation (6.2.14) can easily be integrated over the amplifier length \( L \) to yield

\[
P_{\text{out}}(\tau) = P_{\text{in}}(\tau) \exp[h(\tau)], \tag{6.2.17}
\]

where \( P_{\text{in}}(\tau) \) is the input power and \( h(\tau) \) is the total integrated gain defined as

\[
h(\tau) = \int_0^L g(z, \tau) \, dz. \tag{6.2.18}
\]

If Eq. (6.2.16) is integrated over the amplifier length after replacing \( gP \) by \( \partial P/\partial z \), \( h(\tau) \) satisfies [18]

\[
\frac{dh}{d\tau} = -\frac{1}{E_{\text{sat}}} [P_{\text{out}}(\tau) - P_{\text{in}}(\tau)] = -\frac{P_{\text{in}}(\tau)}{E_{\text{sat}}} (e^h - 1). \tag{6.2.19}
\]

Equation (6.2.19) can easily be solved to obtain \( h(\tau) \). The amplification factor \( G(\tau) \) is related to \( h(\tau) \) as \( G = \exp(h) \) and is given by [1]

\[
G(\tau) = \frac{G_0}{G_0 - (G_0 - 1) \exp[(-E_0(\tau)/E_{\text{sat}})]}, \tag{6.2.20}
\]

where \( G_0 \) is the unsaturated amplifier gain and \( E_0(\tau) = \int_{-\infty}^{\tau} P_{\text{in}}(\tau) \, d\tau \) is the partial energy of the input pulse defined such that \( E_0(\infty) \) equals the input pulse energy \( E_{\text{in}} \).

The solution (6.2.20) shows that the amplifier gain is different for different parts of the pulse. The leading edge experiences the full gain \( G_0 \) as the amplifier is not yet saturated. The trailing edge experiences the least gain since the whole pulse has saturated the amplifier gain. The final value of \( G(\tau) \) after passage of the pulse is obtained from Eq. (6.2.20) by replacing \( E_0(\tau) \) by \( E_{\text{in}} \). The intermediate values of the gain depend on the pulse shape. Figure 6.7 shows the shape dependence of \( G(\tau) \) for super-Gaussian input pulses by using

\[
P_{\text{in}}(t) = P_0 \exp\left[-\left(\frac{t}{\tau_p}\right)^{2m}\right], \tag{6.2.21}
\]

where \( m \) is the shape parameter. The input pulse is Gaussian for \( m = 1 \) but becomes nearly rectangular as \( m \) increases. For comparison purposes, the input energy is held constant for different pulse shapes by choosing \( E_{\text{in}}/E_{\text{sat}} = 0.1 \). The shape dependence of the amplification factor \( G(\tau) \) implies that the output pulse is distorted, and distortion is itself shape dependent.
As seen from Eq. (6.2.15), gain saturation leads to a time-dependent phase shift across the pulse. This phase shift is found by integrating Eq. (6.2.15) over the amplifier length and is given by

$$\phi(\tau) = -\frac{1}{2} \beta_c \int_0^L g(z, \tau) dz = -\frac{1}{2} \beta_c h(\tau) = -\frac{1}{2} \beta_c \ln|G(\tau)|. \quad (6.2.22)$$

Since the pulse modulates its own phase through gain saturation, this phenomenon is referred to as *saturation-induced* self-phase modulation [18]. The frequency chirp is related to the phase derivative as

$$\Delta \nu_c = -\frac{1}{2\pi} \frac{d\phi}{d\tau} = \frac{\beta_c}{4\pi} \frac{dh}{d\tau} = -\frac{\beta_c}{4\pi E_{sat}} P_{in}(\tau) \ln|G(\tau)| - 1], \quad (6.2.23)$$

where Eq. (6.2.19) was used. Figure 6.8 shows the chirp profiles for several input pulse energies when a Gaussian pulse is amplified in a SOA with 30-dB unsaturated gain. The frequency chirp is larger for more energetic pulses simply because gain saturation sets in earlier for such pulses.

Self-phase modulation and the associated frequency chirp can affect lightwave systems considerably. The spectrum of the amplified pulse becomes considerably broad and contains several peaks of different amplitudes [18]. The dominant peak is shifted toward the red side and is broader than the input spectrum. It is also accompanied by one or more satellite peaks. Figure 6.9 shows the expected shape and spectrum of amplified pulses when a Gaussian pulse of energy such that $E_{in}/E_{sat} = 0.1$ is amplified.
by a SOA. The temporal and spectral changes depend on amplifier gain and are quite significant for \( G_0 = 30 \) dB. The experiments performed by using picosecond pulses from mode-locked semiconductor lasers confirm this behavior [18]. In particular, the spectrum of amplified pulses is found to be shifted toward the red side by 50–100 GHz, depending on the amplifier gain. Spectral distortion in combination with the frequency chirp would affect the transmission characteristics when amplified pulses are propagated through optical amplifiers.

It turns out that the frequency chirp imposed by the SOA is opposite in nature compared with that imposed by directly modulated semiconductor lasers. If we also note that the chirp is nearly linear over a considerable portion of the amplified pulse (see Fig. 6.8), it is easy to understand that the amplified pulse would pass through an initial compression stage when it propagates in the anomalous-dispersion region of optical fibers (see Section 2.4.2). Such a compression was observed in an experiment [19] in which 40-ps optical pulses were first amplified in a 1.52-\( \mu \)m SOA and then propagated through 18 km of single-mode fiber with \( \beta_2 = -18 \) ps\(^2\)/km. This compression mechanism can be used to design fiber-optic communication systems in which in-line SOAs are used to compensate simultaneously for both fiber loss and dispersion by operating SOAs in the saturation region so that they impose frequency chirp on the amplified pulse. The basic concept was demonstrated in 1989 in an experiment [20] in which a 16-Gb/s signal was transmitted over 70 km by using an SOA. In the absence of the SOA or when the SOA was operated in the unsaturated regime, the system was dispersion limited to the extent that the signal could not be transmitted over more than 20 km.

The preceding analysis considers a single pulse. In a lightwave system, the signal
6.2. SEMICONDUCTOR OPTICAL AMPLIFIERS

Figure 6.9: (a) Shape and (b) spectrum at the output of a semiconductor optical amplifier with $G_0 = 30$ dB and $\beta_c = 5$ for a Gaussian input pulse of energy $E_{in}/E_{sat} = 0.1$. The dashed curves show for comparison the shape and spectrum of the input pulse.

consists of a random sequence of 1 and 0 bits. If the energy of each 1 bit is large enough to saturate the gain partially, the following bit will experience less gain. The gain will recover partially if the bit 1 is preceded by one or more 0 bits. In effect, the gain of each bit in an SOA depends on the bit pattern. This phenomenon becomes quite problematic for WDM systems in which several pulse trains pass through the amplifier simultaneously. It is possible to implement a gain-control mechanism that keeps the amplifier gain pinned at a constant value. The basic idea is to make the SOA oscillate at a controlled wavelength outside the range of interest (typically below 1.52 µm). Since the gain remains clamped at the threshold value for a laser, the signal is amplified by the same factor for all pulses.

6.2.4 System Applications

The use of SOAs as a preamplifier to the receiver is attractive since it permits monolithic integration of the SOA with the receiver. As seen in Fig. 6.3(c), in this application the signal is optically amplified before it falls on the receiver. The preamplifier boosts the signal to such a high level that the receiver performance is limited by shot noise rather than by thermal noise. The basic idea is similar to the case of avalanche photodiodes (APDs), which amplify the signal in the electrical domain. However, just as APDs add additional noise (see Section 4.4.3), preamplifiers also degrade the SNR through spontaneous-emission noise. A relatively large noise figure of SOAs ($F_n = 5$–$7$ dB) makes them less than ideal as a preamplifier. Nonetheless, they can improve the receiver sensitivity considerably. SOAs can also be used as power amplifiers to boost the transmitter power. It is, however, difficult to achieve powers in excess of 10 mW because of a relatively small value of the output saturation power (~5 mW).

SOAs were used as in-line amplifiers in several system experiments before 1990. In a 1988 experiment, a signal at 1 Gb/s was transmitted over 313 km by using four
cascaded SOAs [21]. SOAs have also been employed to overcome distribution losses in the local-area network (LAN) applications. In one experiment, an SOA was used as a dual-function device [22]. It amplified five channels, but at the same time the SOA was used to monitor the network performance through a baseband control channel. The 100-Mb/s baseband control signal modulated the carrier density of the amplifier, which in turn produced a corresponding electric signal that was used for monitoring.

Although SOAs can be used to amplify several channels simultaneously, they suffer from a fundamental problem related to their relatively fast response. Ideally, the signal in each channel should be amplified by the same amount. In practice, several nonlinear phenomena in SOAs induce interchannel crosstalk, an undesirable feature that should be minimized for practical lightwave systems. Two such nonlinear phenomena are cross-gain saturation and four-wave mixing (FWM). Both of them originate from the stimulated recombination term in Eq. (6.2.6). In the case of multichannel amplification, the power $P$ in this equation is replaced with

$$P = \frac{1}{2} \left| \sum_{j=1}^{M} A_j \exp(-i\omega_j t) + c.c. \right|^2,$$  \hspace{1cm} (6.2.24)

where $c.c.$ stands for the complex conjugate, $M$ is the number of channels, $A_j$ is the amplitude, and $\omega_j$ is the carrier frequency of the $j$th channel. Because of the coherent addition of individual channel fields, Eq. (6.2.24) contains time-dependent terms resulting from beating of the signal in different channels, i.e.,

$$P = \sum_{j=1}^{M} P_j + \sum_{j=1}^{M} \sum_{k \neq j}^{M} 2 \sqrt{P_j P_k} \cos(\Omega_{jk} t + \phi_j - \phi_k),$$  \hspace{1cm} (6.2.25)

where $A_j = \sqrt{P_j} \exp(i\phi_j)$ was assumed together with $\Omega_{jk} = \omega_j - \omega_k$. When Eq. (6.2.25) is substituted in Eq. (6.2.6), the carrier population is also found to oscillate at the beat frequency $\Omega_{jk}$. Since the gain and the refractive index both depend on $N$, they are also modulated at the frequency $\Omega_{jk}$; such a modulation creates gain and index gratings, which induce interchannel crosstalk by scattering a part of the signal from one channel to another. This phenomenon can also be viewed as FWM [16].

The origin of cross-gain saturation is also evident from Eq. (6.2.25). The first term on the right side shows that the power $P$ in Eq. (6.2.7) should be replaced by the total power in all channels. Thus, the gain of a specific channel is saturated not only by its own power but also by the power of neighboring channels, a phenomenon known as cross-gain saturation. It is undesirable in WDM systems since the amplifier gain changes with time depending on the bit pattern of neighboring channels. As a result, the amplified signal appears to fluctuate more or less randomly. Such fluctuations degrade the effective SNR at the receiver. The interchannel crosstalk occurs regardless of the channel spacing. It can be avoided only by reducing the channel powers to low enough values that the SOA operates in the unsaturated regime. Interchannel crosstalk induced by FWM occurs for all WDM lightwave systems irrespective of the modulation format used [23]–[26]. Its impact is most severe for coherent systems because of a relatively small channel spacing [25]. FWM can occur even for widely spaced channels through intraband nonlinearities [17] occurring at fast time scales ($<1$ ps).
6.3. RAMAN AMPLIFIERS

It is clear that SOAs suffer from several drawbacks which make their use as in-line amplifiers impractical. A few among them are polarization sensitivity, interchannel crosstalk, and large coupling losses. The unsuitability of SOAs led to a search for alternative amplifiers during the 1980s, and two types of fiber-based amplifiers using the Raman effect and rare-earth dopants were developed. The following two sections are devoted to these two types of amplifiers. It should be stressed that SOAs have found many other applications. They can be used for wavelength conversion and can act as a fast switch for wavelength routing in WDM networks. They are also being pursued for metropolitan-area networks as a low-cost alternative to fiber amplifiers.

6.3 Raman Amplifiers

A fiber-based Raman amplifier uses stimulated Raman scattering (SRS) occurring in silica fibers when an intense pump beam propagates through it [27]–[29]. The main features of SRS have been discussed in Sections 2.6. SRS differs from stimulated emission in one fundamental aspect. Whereas in the case of stimulated emission an incident photon stimulates emission of another identical photon without losing its energy, in the case of SRS the incident pump photon gives up its energy to create another photon of reduced energy at a lower frequency (inelastic scattering); the remaining energy is absorbed by the medium in the form of molecular vibrations (optical phonons). Thus, Raman amplifiers must be pumped optically to provide gain. Figure 6.10 shows how a fiber can be used as a Raman amplifier. The pump and signal beams at frequencies $\omega_p$ and $\omega_s$ are injected into the fiber through a fiber coupler. The energy is transferred from the pump beam to the signal beam through SRS as the two beams copropagate inside the fiber. The pump and signal beams counterpropagate in the backward-pumping configuration commonly used in practice.

6.3.1 Raman Gain and Bandwidth

The Raman-gain spectrum of silica fibers is shown in Figure 2.18; its broadband nature is a consequence of the amorphous nature of glass. The Raman-gain coefficient $g_R$ is related to the optical gain $g(z)$ as $g = g_R I_p(z)$, where $I_p$ is the pump intensity. In terms of the pump power $P_p$, the gain can be written as

$$g(\omega) = g_R(\omega)(P_p/a_p), \quad (6.3.1)$$
where \( a_p \) is the cross-sectional area of the pump beam inside the fiber. Since \( a_p \) can vary considerably for different types of fibers, the ratio \( g_R/a_p \) is a measure of the Raman-gain efficiency [30]. This ratio is plotted in Fig. 6.11 for three different fibers. A dispersion-compensating fiber (DCF) can be 8 times more efficient than a standard silica fiber (SMF) because of its smaller core diameter. The frequency dependence of the Raman gain is almost the same for the three kinds of fibers as evident from the normalized gain spectra shown in Fig. 6.11. The gain peaks at a Stokes shift of about 13.2 THz. The gain bandwidth \( \Delta \nu_g \) is about 6 THz if we define it as the FWHM of the dominant peak in Fig. 6.11.

The large bandwidth of fiber Raman amplifiers makes them attractive for fiber-optic communication applications. However, a relatively large pump power is required to realize a large amplification factor. For example, if we use Eq. (6.1.7) by assuming operation in the unsaturated region, \( g_L \approx 6.7 \) is required for \( G = 30 \) dB. By using \( g_R = 6 \times 10^{-14} \) m/W at the gain peak at 1.55 \( \mu \)m and \( a_p = 50 \) \( \mu \)m\(^2\), the required pump power is more than 5 W for 1-km-long fiber. The required power can be reduced for longer fibers, but then fiber losses must be included. In the following section we discuss the theory of Raman amplifiers including both fiber losses and pump depletion.

### 6.3.2 Amplifier Characteristics

It is necessary to include the effects of fiber losses because of a long fiber length required for Raman amplifiers. Variations in the pump and signal powers along the amplifier length can be studied by solving the two coupled equations given in Section 2.6.1. In the case of forward pumping, these equations take the form

\[
dP_s/dz = -\alpha_s P_s + (g_R/a_p)P_p P_s, \quad (6.3.2)
\]
\[
dP_p/dz = -\alpha_p P_p - (\omega_p/\omega_s)(g_R/a_p)P_s P_p, \quad (6.3.3)
\]

where \( \alpha_s \) and \( \alpha_p \) represent fiber losses at the signal and pump frequencies \( \omega_s \) and \( \omega_p \), respectively. The factor \( \omega_p/\omega_s \) results from different energies of pump and signal photons and disappears if these equations are written in terms of photon numbers.
Consider first the case of small-signal amplification for which pump depletion can be neglected [the last term in Eq. (6.3.3)]. Substituting $P_p(z) = P_p(0) \exp(-\alpha_p z)$ in Eq. (6.3.2), the signal power at the output of an amplifier of length $L$ is given by

$$P_s(L) = P_s(0) \exp(g_R P_0 L_{\text{eff}} / a_p - \alpha_s L),$$

(6.3.4)

where $P_0 = P_p(0)$ is the input pump power and $L_{\text{eff}}$ is defined as

$$L_{\text{eff}} = \left[1 - \exp(-\alpha_p L)\right] / \alpha_p.$$

(6.3.5)

Because of fiber losses at the pump wavelength, the effective length of the amplifier is less than the actual length $L$; $L_{\text{eff}} \approx 1 / \alpha_p$ for $\alpha_p L \gg 1$. Since $P_s(L) = P_s(0) \exp(-\alpha_s L)$ in the absence of Raman amplification, the amplifier gain is given by

$$G_A = \frac{P_s(L)}{P_s(0) \exp(-\alpha_s L)} = \exp(g_0 L),$$

(6.3.6)

where the small-signal gain $g_0$ is defined as

$$g_0 = g_R \left(\frac{P_0}{a_p}\right) \left(\frac{L_{\text{eff}}}{L}\right) \approx \frac{g_R P_0}{a_p \alpha_p L}.$$

(6.3.7)

The last relation holds for $\alpha_p L \gg 1$. The amplification factor $G_A$ becomes length independent for large values of $\alpha_p L$. Figure 6.12 shows variations of $G_A$ with $P_0$ for several values of input signal powers for a 1.3-km-long Raman amplifier operating at 1.064 µm and pumped at 1.017 µm. The amplification factor increases exponentially with $P_0$ initially but then starts to deviate for $P_0 > 1$ W because of gain saturation. Deviations become larger with an increase in $P_s(0)$ as gain saturation sets in earlier along the amplifier length. The solid lines in Fig. 6.12 are obtained by solving Eqs. (6.3.2) and (6.3.3) numerically to include pump depletion.

The origin of gain saturation in Raman amplifiers is quite different from SOAs. Since the pump supplies energy for signal amplification, it begins to deplete as the signal power $P_s$ increases. A decrease in the pump power $P_p$ reduces the optical gain as seen from Eq. (6.3.1). This reduction in gain is referred to as gain saturation. An approximate expression for the saturated amplifier gain $G_s$ can be obtained assuming $\alpha_s = \alpha_p$ in Eqs. (6.3.2) and (6.3.3). The result is given by [29]

$$G_s = \frac{1 + r_0}{r_0 + G_A^{-1+ r_0}}, \quad r_0 = \frac{a_p P_s(0)}{a_s P_p(0)}.$$  

(6.3.8)

Figure 6.13 shows the saturation characteristics by plotting $G_s / G_A$ as a function of $G_A r_0$ for several values of $G_A$. The amplifier gain is reduced by 3 dB when $G_A r_0 \approx 1$. This condition is satisfied when the power of the amplified signal becomes comparable to the input pump power $P_0$. In fact, $P_0$ is a good measure of the saturation power. Since typically $P_0 \sim 1$ W, the saturation power of fiber Raman amplifiers is much larger than that of SOAs. As typical channel powers in a WDM system are $\sim 1$ mW, Raman amplifiers operate in the unsaturated or linear regime, and Eq. (6.3.7) can be used in place of Eq. (6.3.8).
Noise in Raman amplifiers stems from spontaneous Raman scattering. It can be included in Eq. (6.3.2) by replacing $P_s$ in the last term with $P_s + P_{sp}$, where $P_{sp} = 2n_{sp}h\nu_s\Delta \nu_R$ is the total spontaneous Raman power over the entire Raman-gain bandwidth $\Delta \nu_R$. The factor of 2 accounts for the two polarization directions. The factor $n_{sp}(\Omega)$ equals $\left[1 - \exp(-\hbar\Omega_s/\kappa_B T)\right]^{-1}$, where $\kappa_B T$ is the thermal energy at room temperature (about 25 meV). In general, the added noise is much smaller for Raman amplifiers because of the distributed nature of the amplification.

### 6.3.3 Amplifier Performance

As seen in Fig. 6.12, Raman amplifiers can provide 20-dB gain at a pump power of about 1 W. For the optimum performance, the frequency difference between the pump and signal beams should correspond to the peak of the Raman gain in Fig. 6.11 (occurring at about 13 THz). In the near-infrared region, the most practical pump source is a diode-pumped Nd:YAG laser operating at 1.06 $\mu$m. For such a pump laser, the maximum gain occurs for signal wavelengths near 1.12 $\mu$m. However, the wavelengths of most interest for fiber-optic communication systems are near 1.3 and 1.5 $\mu$m. A
Nd:YAG laser can still be used if a higher-order Stokes line, generated through cascaded SRS, is used as a pump. For instance, the third-order Stokes line at 1.24 µm can act as a pump for amplifying the 1.3-µm signal. Amplifier gains of up to 20 dB were measured in 1984 with this technique [32]. An early application of Raman amplifiers was as a preamplifier for improving the receiver sensitivity [33].

The broad bandwidth of Raman amplifiers is useful for amplifying several channels simultaneously. As early as 1988 [34], signals from three DFB semiconductor lasers operating in the range 1.57–1.58 µm were amplified simultaneously using a 1.47-µm pump. This experiment used a semiconductor laser as a pump source. An amplifier gain of 5 dB was realized at a pump power of only 60 mW. In another interesting experiment [35], a Raman amplifier was pumped by a 1.55-µm semiconductor laser whose output was amplified using an erbium-doped fiber amplifier. The 140-ns pump pulses had 1.4 W peak power at the 1-kHz repetition rate and were capable of amplifying 1.66-µm signal pulses by more than 23 dB through SRS in a 20-km-long dispersion-shifted fiber. The 200 mW peak power of 1.66-µm pulses was large enough for their use for optical time-domain reflection measurements commonly used for supervising and maintaining fiber-optic networks [36].

The use of Raman amplifiers in the 1.3-µm spectral region has also attracted attention [37]–[40]. However, a 1.24-µm pump laser is not readily available. Cascaded SRS can be used to generate the 1.24-µm pump light. In one approach, three pairs of fiber gratings are inserted within the fiber used for Raman amplification [37]. The Bragg wavelengths of these gratings are chosen such that they form three cavities for three Raman lasers operating at wavelengths 1.117, 1.175, and 1.24 µm that correspond to first-, second-, and third-order Stokes lines of the 1.06-µm pump. All three lasers are pumped by using a diode-pumped Nd-fiber laser through cascaded SRS. The 1.24-µm
laser then pumps the Raman amplifier and amplifies a 1.3-µm signal. The same idea of cascaded SRS was used to obtain 39-dB gain at 1.3 µm by using WDM couplers in place of fiber gratings [38]. Such 1.3-µm Raman amplifiers exhibit high gains with a low noise figure (about 4 dB) and are also suitable as an optical preamplifier for high-speed optical receivers. In a 1996 experiment, such a receiver yielded the sensitivity of 151 photons/bit at a bit rate of 10 Gb/s [39]. The 1.3-µm Raman amplifiers can also be used to upgrade the capacity of existing fiber links from 2.5 to 10 Gb/s [40].

Raman amplifiers are called lumped or distributed depending on their design. In the lumped case, a discrete device is made by spooling 1–2 km of a especially prepared fiber that has been doped with Ge or phosphorus for enhancing the Raman gain. The fiber is pumped at a wavelength near 1.45 µm for amplification of 1.55-µm signals.

In the case of distributed Raman amplification, the same fiber that is used for signal transmission is also used for signal amplification. The pump light is often injected in the backward direction and provides gain over relatively long lengths (>20 km). The main drawback in both cases from the system standpoint is that high-power lasers are required for pumping. Early experiments often used a tunable color-center laser as a pump; such lasers are too bulky for system applications. For this reason, Raman amplifiers were rarely used during the 1990s after erbium-doped fiber amplifiers became available. The situation changed by 2000 with the availability of compact high-power semiconductor and fiber lasers.

The phenomenon that limits the performance of distributed Raman amplifiers most turns out to be Rayleigh scattering [41]–[45]. As discussed in Section 2.5, Rayleigh scattering occurs in all fibers and is the fundamental loss mechanism for them. A small part of light is always backscattered because of this phenomenon. Normally, this Rayleigh backscattering is negligible. However, it can be amplified over long lengths in fibers with distributed gain and affects the system performance in two ways. First, a part of backward propagating noise appears in the forward direction, enhancing the overall noise. Second, double Rayleigh scattering of the signal creates a crosstalk component in the forward direction. It is this Rayleigh crosstalk, amplified by the distributed Raman gain, that becomes the major source of power penalty. The fraction of signal power propagating in the forward direction after double Rayleigh scattering is the Rayleigh crosstalk. This fraction is given by [43]

$$f_{\text{DRS}} = r_s^2 \int_0^L dz_1 G^{-2}(z_1) \int_{z_1}^L G^2(z_2) \, dz_2,$$

where $r_s \sim 10^{-4}$ km$^{-1}$ is the Rayleigh scattering coefficient and $G(z)$ is the Raman gain at a distance $z$ in the backward-pumping configuration for an amplifier of length $L$. The crosstalk level can exceed 1% (~20-dB crosstalk) for $L > 80$ km and $G(L) > 10$. Since this crosstalk accumulates over multiple amplifiers, it can lead to large power penalties for undersea lightwave systems with long lengths.

Raman amplifiers can work at any wavelength as long as the pump wavelength is suitably chosen. This property, coupled with their wide bandwidth, makes Raman amplifiers quite suitable for WDM systems. An undesirable feature is that the Raman gain is somewhat polarization sensitive. In general, the gain is maximum when the signal and pump are polarized along the same direction but is reduced when they are
orthogonally polarized. The polarization problem can be solved by pumping a Raman amplifier with two orthogonally polarized lasers. Another requirement for WDM systems is that the gain spectrum be relatively uniform over the entire signal bandwidth so that all channels experience the same gain. In practice, the gain spectrum is flattened by using several pumps at different wavelengths. Each pump creates the gain that mimics the spectrum shown in Fig. 6.11. The superposition of several such spectra then creates relatively flat gain over a wide spectral region. Bandwidths of more than 100 nm have been realized using multiple pump lasers [46]–[48].

The design of broadband Raman amplifiers suitable for WDM applications requires consideration of several factors. The most important among them is the inclusion of pump–pump interactions. In general, multiple pump beams are also affected by the Raman gain, and some power from each short-wavelength pump is invariably transferred to long-wavelength pumps. An appropriate model that includes pump interactions, Rayleigh backscattering, and spontaneous Raman scattering considers each frequency component separately and solves the following set of coupled equations [48]:

\[
\begin{align*}
\frac{dP_f(\nu)}{dz} &= \int_{\nu < \mu} g_R(\mu - \nu) a_\nu^{-1} [P_f(\mu) + P_b(\mu)][P_f(\nu) + 2h v n_{sp}(\mu - \nu)] d\mu \\
&- \int_{\mu > \nu} g_R(\nu - \mu) a_\mu^{-1} [P_f(\mu) + P_b(\mu)][P_f(\nu) + 2h v n_{sp}(\nu - \mu)] d\mu \\
&- \alpha(\nu) P_f(\nu) + r_s P_b(\nu)
\end{align*}
\]

(6.3.10)

where \( \mu \) and \( \nu \) denote optical frequencies, \( n_{sp}(\Omega) = [1 - \exp(-\hbar \Omega/k_B T)]^{-1} \), and the subscripts \( f \) and \( b \) denote forward- and backward-propagating waves, respectively. In this equation, the first and second terms account for the Raman-induced power transfer into and out of each frequency band. Fiber losses and Rayleigh backscattering are included through the third and fourth terms, respectively. The noise induced by spontaneous Raman scattering is included by the temperature-dependent factor in the two integrals. A similar equation can be written for the backward-propagating waves.

To design broadband Raman amplifiers, the entire set of such equations is solved numerically to find the channel gains, and input pump powers are adjusted until the gain is nearly the same for all channels. Figure 6.14 shows an example of the gain spectrum measured for a Raman amplifier made by pumping a 25-km-long dispersion-shifted fiber with 12 diode lasers. The frequencies and power levels of the pump lasers, required to achieve a nearly flat gain profile, are also shown. Notice that all power levels are under 100 mW. The amplifier provides about 10.5 dB gain over an 80-nm bandwidth with a ripple of less than 0.1 dB. Such an amplifier is suitable for dense WDM systems covering both the C and L bands. Several experiments have used broadband Raman amplifiers to demonstrate transmission over long distances at high bit rates. In one 3-Tb/s experiment, 77 channels, each operating at 42.7 Gb/s, were transmitted over 1200 km by using the C and L bands simultaneously [49].

Several other nonlinear processes can provide gain inside silica fibers. An example is provided by the parametric gain resulting from FWM [29]. The resulting fiber amplifier is called a parametric amplifier and can have a gain bandwidth larger than 100 nm. Parametric amplifiers require a large pump power (typically >1 W) that may be reduced using fibers with high nonlinearities. They also generate a phase-conjugated
signal that can be useful for dispersion compensation (see Section 7.7). Fiber amplifiers can also be made using stimulated Brillouin scattering (SBS) in place of SRS [29]. The operating mechanism behind Brillouin amplifiers is essentially the same as that for fiber Raman amplifiers in the sense that both amplifiers are pumped backward and provide gain through a scattering process. Despite this formal similarity, Brillouin amplifiers are rarely used in practice because their gain bandwidth is typically below 100 MHz. Moreover, as the Stokes shift for SBS is \( \sim 10 \) GHz, pump and signal wavelengths nearly coincide. These features render Brillouin amplifiers unsuitable for WDM lightwave systems although they can be exploited for other applications.

### 6.4 Erbium-Doped Fiber Amplifiers

An important class of fiber amplifiers makes use of rare-earth elements as a gain medium by doping the fiber core during the manufacturing process (see Section 2.7). Although doped-fiber amplifiers were studied as early as 1964 [50], their use became practical only 25 years later, after the fabrication and characterization techniques were perfected [51]. Amplifier properties such as the operating wavelength and the gain bandwidth are determined by the dopants rather than by the silica fiber, which plays the role of a host medium. Many different rare-earth elements, such as erbium, holmium, neodymium, samarium, thulium, and ytterbium, can be used to realize fiber amplifiers operating at different wavelengths in the range 0.5–3.5 \( \mu \text{m} \). Erbium-doped fiber amplifiers (EDFAs) have attracted the most attention because they operate in the wavelength region near 1.55 \( \mu \text{m} \) [52]–[56]. Their deployment in WDM systems after 1995 revolutionized the field of fiber-optic communications and led to lightwave systems with capacities exceeding 1 Tb/s. This section focuses on the main characteristics of EDFAs.
Figure 6.15: (a) Energy-level diagram of erbium ions in silica fibers; (b) absorption and gain spectra of an EDFA whose core was codoped with germania. (After Ref. [64]; ©1991 IEEE; reprinted with permission.)

6.4.1 Pumping Requirements

The design of an EDFA looks similar to that shown in Fig. 6.10 with the main difference that the fiber core contains erbium ions (Er$^{3+}$). Pumping at a suitable wavelength provides gain through population inversion. The gain spectrum depends on the pumping scheme as well as on the presence of other dopants, such as germania and alumina, within the fiber core. The amorphous nature of silica broadens the energy levels of Er$^{3+}$ into bands. Figure 6.15(a) shows a few energy levels of Er$^{3+}$ in silica glasses. Many transitions can be used to pump an EDFA. Early experiments used the visible radiation emitted from argon-ion, Nd:YAG, or dye lasers even though such pumping schemes are relatively inefficient. From a practical standpoint the use of semiconductor lasers is preferred.

Efficient EDFA pumping is possible using semiconductor lasers operating near 0.98- and 1.48-$\mu$m wavelengths. Indeed, the development of such pump lasers was fueled with the advent of EDFAs. It is possible to realize 30-dB gain with only 10–15 mW of absorbed pump power. Efficiencies as high as 11 dB/mW were achieved by 1990 with 0.98-$\mu$m pumping [57]. The pumping transition $^4I_{15/2} \rightarrow ^4I_{9/2}$ can use high-power GaAs lasers, and the pumping efficiency of about 1 dB/mW has been obtained at 820 nm [58]. The required pump power can be reduced by using silica fibers doped with aluminum and phosphorus or by using fluorophosphate fibers [59]. With the availability of visible semiconductor lasers, EDFAs can also be pumped in the wavelength range 0.6–0.7 $\mu$m. In one experiment [60], 33-dB gain was realized at 27 mW of pump power obtained from an AlGaInP laser operating at 670 nm. The pumping efficiency was as high as 3 dB/mW at low pump powers. Most EDFAs use 980-nm pump lasers as such lasers are commercially available and can provide more than 100 mW of pump...
power. Pumping at 1480 nm requires longer fibers and higher powers because it uses the tail of the absorption band shown in Fig. 6.15(b).

EDFAs can be designed to operate in such a way that the pump and signal beams propagate in opposite directions, a configuration referred to as backward pumping to distinguish it from the forward-pumping configuration shown in Fig. 6.10. The performance is nearly the same in the two pumping configurations when the signal power is small enough for the amplifier to remain unsaturated. In the saturation regime, the power-conversion efficiency is generally better in the backward-pumping configuration [61], mainly because of the important role played by the amplified spontaneous emission (ASE). In the bidirectional pumping configuration, the amplifier is pumped in both directions simultaneously by using two semiconductor lasers located at the two fiber ends. This configuration requires two pump lasers but has the advantage that the population inversion, and hence the small-signal gain, is relatively uniform along the entire amplifier length.

### 6.4.2 Gain Spectrum

The gain spectrum shown in Fig. 6.15 is the most important feature of an EDFA as it determines the amplification of individual channels when a WDM signal is amplified. The shape of the gain spectrum is affected considerably by the amorphous nature of silica and by the presence of other codopants within the fiber core such as germania and alumina [62]–[64]. The gain spectrum of erbium ions alone is homogeneously broadened; its bandwidth is determined by the dipole relaxation time $T_2$ in accordance with Eq. (6.1.2). However, the spectrum is considerably broadened in the presence of randomly located silica molecules. Structural disorders lead to inhomogeneous broadening of the gain spectrum, whereas Stark splitting of various energy levels is responsible for homogeneous broadening. Mathematically, the gain $g(\omega)$ in Eq. (6.1.2) should be averaged over the distribution of atomic transition frequencies $\omega_0$ such that the effective gain is given by

$$g_{\text{eff}}(\omega) = \int_{-\infty}^{\infty} g(\omega, \omega_0) f(\omega_0) d\omega_0,$$

where $f(\omega_0)$ is the distribution function whose form also depends on the presence of other dopants within the fiber core.

Figure 6.15(b) shows the gain and absorption spectra of an EDFA whose core was doped with germania [64]. The gain spectrum is quite broad and has a double-peak structure. The addition of alumina to the fiber core broadens the gain spectrum even more. Attempts have been made to isolate the contributions of homogeneous and inhomogeneous broadening through measurements of spectral hole burning. For germania-doped EDFAs the contributions of homogeneous and inhomogeneous broadening are relatively small [63]. In contrast, the gain spectrum of aluminosilicate glasses has roughly equal contributions from homogeneous and inhomogeneous broadening mechanisms. The gain bandwidth of such EDFAs typically exceeds 35 nm.

The gain spectrum of EDFAs can vary from amplifier to amplifier even when core composition is the same because it also depends on the amplifier length. The reason is that the gain depends on both the absorption and emission cross sections having different spectral characteristics. The local inversion or local gain varies along the fiber
length because of pump power variations. The total gain is obtained by integrating over the amplifier length. This feature can be used to realize EDFAs that provide amplification in the L band covering the spectral region 1570–1610 nm. The wavelength range over which an Edfa can provide nearly constant gain is of primary interest for WDM systems. This issue is discussed later in this section.

6.4.3 Simple Theory

The gain of an EDFA depends on a large number of device parameters such as erbium-ion concentration, amplifier length, core radius, and pump power [64]–[68]. A three-level rate-equation model commonly used for lasers [1] can be adapted for EDFAs. It is sometimes necessary to add a fourth level to include the excited-state absorption. In general, the resulting equations must be solved numerically.

Considerable insight can be gained by using a simple two-level model that is valid when ASE and excited-state absorption are negligible. The model assumes that the top level of the three-level system remains nearly empty because of a rapid transfer of the pumped population to the excited state. It is, however, important to take into account the different emission and absorption cross sections for the pump and signal fields. The population densities of the two states, \( N_1 \) and \( N_2 \), satisfy the following two rate equations [55]:

\[
\frac{\partial N_2}{\partial t} = (\sigma_p^a N_1 - \sigma_p^e N_2) \phi_p + (\sigma_s^a N_1 - \sigma_s^e N_2) \phi_s - \frac{N_2}{T_1}, \tag{6.4.2}
\]

\[
\frac{\partial N_1}{\partial t} = (\sigma_p^e N_2 - \sigma_p^a N_1) \phi_p + (\sigma_s^e N_2 - \sigma_s^a N_1) \phi_s + \frac{N_2}{T_1}. \tag{6.4.3}
\]

where \( \sigma_j^a \) and \( \sigma_j^e \) are the absorption and emission cross sections at the frequency \( \omega_j \) with \( j = p, s \). Further, \( T_1 \) is the spontaneous lifetime of the excited state (about 10 ms for EDFAs). The quantities \( \phi_p \) and \( \phi_s \) represent the photon flux for the pump and signal waves, defined such that \( \phi_j = P_j / (a_j h \nu_j) \), where \( P_j \) is the optical power, \( \sigma_j \) is the transition cross section at the frequency \( \nu_j \), and \( a_j \) is the cross-sectional area of the fiber mode for \( j = p, s \).

The pump and signal powers vary along the amplifier length because of absorption, stimulated emission, and spontaneous emission. If the contribution of spontaneous emission is neglected, \( P_s \) and \( P_p \) satisfy the simple equations

\[
\frac{\partial P_s}{\partial z} = \Gamma_s (\sigma_s^e N_2 - \sigma_s^a N_1) P_s - \alpha P_s, \tag{6.4.4}
\]

\[
\frac{\partial P_p}{\partial z} = \Gamma_p (\sigma_p^e N_2 - \sigma_p^a N_1) P_p - \alpha P_p, \tag{6.4.5}
\]

where \( \alpha \) and \( \alpha' \) take into account fiber losses at the signal and pump wavelengths, respectively. These losses can be neglected for typical amplifier lengths of 10–20 m. However, they must be included in the case of distributed amplification discussed later. The confinement factors \( \Gamma_s \) and \( \Gamma_p \) account for the fact that the doped region within the core provides the gain for the entire fiber mode. The parameter \( s = \pm 1 \) in Eq. (6.4.5)
Figure 6.16: Small-signal gain as a function of (a) pump power and (b) amplifier length for an EDFA assumed to be pumped at 1.48 $\mu$m. (After Ref. [64]; ©1991 IEEE; reprinted with permission.)

depending on the direction of pump propagation; $s = -1$ in the case of a backward-propagating pump.

Equations (6.4.2)–(6.4.5) can be solved analytically, in spite of their complexity, after some justifiable approximations [65]. For lumped amplifiers, the fiber length is short enough that both $\alpha$ and $\alpha'$ can be set to zero. Noting that $N_1 + N_2 = N_t$ where $N_t$ is the total ion density, only one equation, say Eq. (6.4.2) for $N_2$, need be solved. Noting again that the absorption and stimulated-emission terms in the field and population equations are related, the steady-state solution of Eq. (6.4.2), obtained by setting the time derivative to zero, can be written as

$$N_2(z) = -\frac{T_1}{a_d h v_s} \frac{\partial P_s}{\partial z} - \frac{s T_1}{a_d h v_p} \frac{\partial P_p}{\partial z},$$

(6.4.6)

where $a_d = \Gamma_s a_s = \Gamma_p a_p$ is the cross-sectional area of the doped portion of the fiber core. Substituting this solution into Eqs. (6.4.4) and (6.4.5) and integrating them over the fiber length, the powers $P_s$ and $P_p$ at the fiber output can be obtained in an analytical form. This model has been extended to include the ASE propagation in both the forward and backward directions [68].

The total amplifier gain $G$ for an EDFA of length $L$ is obtained using

$$G = \Gamma_s \exp \left[ \int_0^L \left( \sigma_s^e N_2 - \sigma_s^a N_1 \right) dz \right],$$

(6.4.7)

where $N_1 = N_t - N_2$ and $N_2$ is given by Eq. (6.4.6). Figure 6.16 shows the small-signal gain at 1.55 $\mu$m as a function of the pump power and the amplifier length by using typical parameter values. For a given amplifier length $L$, the amplifier gain initially increases exponentially with the pump power, but the increase becomes much smaller when the pump power exceeds a certain value [corresponding to the “knee” in Fig. 6.16(a)]. For a given pump power, the amplifier gain becomes maximum at an optimum value of $L$ and drops sharply when $L$ exceeds this optimum value. The reason is that the latter portion of the amplifier remains unpumped and absorbs the amplified signal.
Since the optimum value of $L$ depends on the pump power $P_p$, it is necessary to choose both $L$ and $P_p$ appropriately. Figure 6.16(b) shows that a 35-dB gain can be realized at a pump power of 5 mW for $L = 30$ m and 1.48-µm pumping. It is possible to design amplifiers such that high gain is obtained for amplifier lengths as short as a few meters. The qualitative features shown in Fig. 6.16 are observed in all EDFAs; the agreement between theory and experiment is generally quite good [67]. The saturation characteristics of EDFAs are similar to those shown in Figs. 6.13 for Raman amplifiers. In general, the output saturation power is smaller than the output pump power expected in the absence of signal. It can vary over a wide range depending on the EDFA design, with typical values $\sim 10$ mW. For this reason the output power levels of EDFAs are generally limited to below 100 mW, although powers as high as 250 mW have been obtained with a proper design [69].

The foregoing analysis assumes that both pump and signal waves are in the form of CW beams. In practice, EDFAs are pumped by using CW semiconductor lasers, but the signal is in the form of a pulse train (containing a random sequence of 1 and 0 bits), and the duration of individual pulses is inversely related to the bit rate. The question is whether all pulses experience the same gain or not. As discussed in Section 6.2, the gain of each pulse depends on the preceding bit pattern for SOAs because an SOA can respond on time scales of 100 ps or so. Fortunately, the gain remains constant with time in an EDFA for even microsecond-long pulses. The reason is related to a relatively large value of the fluorescence time associated with the excited erbium ions ($T_1 \sim 10$ ms). When the time scale of signal-power variations is much shorter than $T_1$, erbium ions are unable to follow such fast variations. As single-pulse energies are typically much below the saturation energy ($\sim 10 \mu$J), EDFAs respond to the average power. As a result, gain saturation is governed by the average signal power, and amplifier gain does not vary from pulse to pulse even for a WDM signal.

In some applications such as packet-switched networks, signal power may vary on a time scale comparable to $T_1$. Amplifier gain in that case is likely to become time dependent, an undesirable feature from the standpoint of system performance. A gain-control mechanism that keeps the amplifier gain pinned at a constant value consists of making the EDFA oscillate at a controlled wavelength outside the range of interest (typically below 1.5 µm). Since the gain remains clamped at the threshold value for a laser, the signal is amplified by the same factor despite variations in the signal power. In one implementation of this scheme, an EDFA was forced to oscillate at 1.48 µm by fabricating two fiber Bragg gratings acting as high-reflectivity mirrors at the two ends of the amplifier [70].

### 6.4.4 Amplifier Noise

Amplifier noise is the ultimate limiting factor for system applications [71]–[74]. For a lumped EDFA, the impact of ASE is quantified through the noise figure $F_n$ given by $F_n = 2n_{sp}$. The spontaneous emission factor $n_{sp}$ depends on the relative populations $N_1$ and $N_2$ of the ground and excited states as $n_{sp} = N_2/(N_2 - N_1)$. Since EDFAs operate on the basis of a three-level pumping scheme, $N_1 \neq 0$ and $n_{sp} > 1$. Thus, the noise figure of EDFAs is expected to be larger than the ideal value of 3 dB.
The spontaneous-emission factor can be calculated for an EDFA by using the rate-equation model discussed earlier. However, one should take into account the fact that both $N_1$ and $N_2$ vary along the fiber length because of their dependence on the pump and signal powers; hence $n_{sp}$ should be averaged along the amplifier length. As a result, the noise figure depends both on the amplifier length $L$ and the pump power $P_p$, just as the amplifier gain does. Figure 6.17(a) shows the variation of $F_n$ with the amplifier length for several values of $P_p/P_{sat}$ when a 1.53-µm signal is amplified with an input power of 1 mW. The amplifier gain under the same conditions is also shown in Fig. 6.17(b). The results show that a noise figure close to 3 dB can be obtained for a high-gain amplifier pumped such that $P_p \gg P_{sat}$ [71].

The experimental results confirm that $F_n$ close to 3 dB is possible in EDFAs. A noise figure of 3.2 dB was measured in a 30-m-long EDFA pumped at 0.98 µm with 11 mW of power [72]. A similar value was found for another EDFA pumped with only 5.8 mW of pump power at 0.98 µm [73]. In general, it is difficult to achieve high gain, low noise, and high pumping efficiency simultaneously. The main limitation is imposed by the ASE traveling backward toward the pump and depleting the pump power. Incorporation of an internal isolator alleviates this problem to a large extent. In one implementation, 51-dB gain was realized with a 3.1-dB noise figure at a pump power of only 48 mW [75].

The measured values of $F_n$ are generally larger for EDFAs pumped at 1.48 µm. A noise figure of 4.1 dB was obtained for a 60-m-long EDFA when pumped at 1.48 µm with 24 mW of pump power [72]. The reason for a larger noise figure for 1.48-µm pumped EDFAs can be understood from Fig. 6.17(a), which shows that the pump level and the excited level lie within the same band for 1.48-µm pumping. It is difficult to achieve complete population inversion ($N_1 \approx 0$) under such conditions. It is nonetheless possible to realize $F_n < 3.5$ dB for pumping wavelengths near 1.46 µm.
6.4. ERBIUM-DOPED FIBER AMPLIFIERS

Relatively low noise levels of EDFAs make them an ideal choice for WDM lightwave systems. In spite of low noise, the performance of long-haul fiber-optic communication systems employing multiple EDFAs is often limited by the amplifier noise. The noise problem is particularly severe when the system operates in the anomalous-dispersion region of the fiber because a nonlinear phenomenon known as the modulation instability [29] enhances the amplifier noise [76] and degrades the signal spectrum [77]. Amplifier noise also introduces timing jitter. These issues are discussed later in this chapter.

6.4.5 Multichannel Amplification

The bandwidth of EDFAs is large enough that they have proven to be the optical amplifier of choice for WDM applications. The gain provided by them is nearly polarization insensitive. Moreover, the interchannel crosstalk that cripples SOAs because of the carrier-density modulation occurring at the channel spacing does not occur in EDFAs. The reason is related to the relatively large value of $T_1$ (about 10 ms) compared with the carrier lifetime in SOAs (<1 ns). The sluggish response of EDFAs ensures that the gain cannot be modulated at frequencies much larger than 10 kHz.

A second source of interchannel crosstalk is cross-gain saturation occurring because the gain of a specific channel is saturated not only by its own power (self-saturation) but also by the power of neighboring channels. This mechanism of crosstalk is common to all optical amplifiers including EDFAs [78]–[80]. It can be avoided by operating the amplifier in the unsaturated regime. Experimental results support this conclusion. In a 1989 experiment [78], negligible power penalty was observed when an EDFA was used to amplify two channels operating at 2 Gb/s and separated by 2 nm as long as the channel powers were low enough to avoid the gain saturation.

The main practical limitation of an EDFA stems from the spectral nonuniformity of the amplifier gain. Even though the gain spectrum of an EDFA is relatively broad, as seen in Fig. 6.15, the gain is far from uniform (or flat) over a wide wavelength range. As a result, different channels of a WDM signal are amplified by different amounts. This problem becomes quite severe in long-haul systems employing a cascaded chain of EDFAs. The reason is that small variations in the amplifier gain for individual channels grow exponentially over a chain of in-line amplifiers if the gain spectrum is the same for all amplifiers. Even a 0.2-dB gain difference grows to 20 dB over a chain of 100 in-line amplifiers, making channel powers vary by a factor of 100, an unacceptable variation range in practice. To amplify all channels by nearly the same amount, the double-peak nature of the EDFA gain spectrum forces one to pack all channels near one of the gain peaks. In a simple approach, input powers of different channels were adjusted to reduce power variations at the receiver to an acceptable level [81]. This technique may work for a small number of channels but becomes unsuitable for dense WDM systems.

The entire bandwidth of 35–40 nm can be used if the gain spectrum is flattened by introducing wavelength-selective losses through an optical filter. The basic idea behind gain flattening is quite simple. If an optical filter whose transmission losses mimic the gain profile (high in the high-gain region and low in the low-gain region) is inserted after the doped fiber, the output power will become constant for all channels.
Although fabrication of such a filter is not simple, several gain-flattening techniques have been developed [55]. For example, thin-film interference filters, Mach–Zehnder filters, acousto-optic filters, and long-period fiber gratings have been used for flattening the gain profile and equalizing channel gains [82]–[84].

The gain-flattening techniques can be divided into active and passive categories. Most filter-based methods are passive in the sense that channel gains cannot be adjusted in a dynamic fashion. The location of the optical filter itself requires some thought because of high losses associated with it. Placing it before the amplifier increases the noise while placing it after the amplifier reduces the output power. Often a two-stage configuration shown in Fig. 6.18 is used. The second stage acts as a power amplifier while the noise figure is mostly determined by the first stage whose noise is relatively low because of its low gain. A combination of several long-period fiber gratings acting as the optical filter in the middle of two stages resulted by 1977 in an EDFA whose gain was flat to within 1 dB over the 40-nm bandwidth in the wavelength range of 1530–1570 nm [85].

Ideally, an optical amplifier should provide the same gain for all channels under all possible operating conditions. This is not the case in general. For instance, if the number of channels being transmitted changes, the gain of each channel will change since it depends on the total signal power because of gain saturation. The active control of channel gains is thus desirable for WDM applications. Many techniques have been developed for this purpose. The most commonly used technique stabilizes the gain dynamically by incorporating within the amplifier a laser that operates outside the used bandwidth. Such devices are called gain-clamped EDFAs (as their gain is clamped by a built-in laser) and have been studied extensively [86]–[91].

WDM lightwave systems capable of transmitting more than 80 channels appeared by 1998. Such systems use the C and L bands simultaneously and need uniform amplifier gain over a bandwidth exceeding 60 nm. Moreover, the use of the L band requires optical amplifiers capable of providing gain in the wavelength range 1570–1610 nm. It turns out that EDFAs can provide gain over this wavelength range, with a suitable design. An L-band EDFA requires long fiber lengths (>100 m) to keep the inversion level relatively low. Figure 6.19 shows an L-band amplifier with a two-stage
6.4. ERBIUM-DOPED FIBER AMPLIFIERS

Figure 6.19: Schematic of an L-band EDFA providing uniform gain over the 1570–1610-nm bandwidth with a two-stage design. (After Ref. [92]; ©1999 IEEE; reprinted with permission.)

design [92]. The first stage is pumped at 980 nm and acts as a traditional EDFA (fiber length 20–30 m) capable of providing gain in the range 1530–1570 nm. In contrast, the second stage has 200-m-long doped fiber and is pumped bidirectionally using 1480-nm lasers. An optical isolator between the two stages passes the ASE from the first stage to the second stage (necessary for pumping the second stage) but blocks the backward-propagating ASE from entering the first stage. Such cascaded, two-stage amplifiers can provide flat gain over a wide bandwidth while maintaining a relatively low noise level. As early as 1996, flat gain to within 0.5 dB was realized over the wavelength range of 1544–1561 nm [93]. The second EDFA was codoped with ytterbium and phosphorus and was optimized such that it acted as a power amplifier. Since then, EDFAs providing flat gain over the entire C and L bands have been made [55]. Raman amplification can also be used for the L band. Combining Raman amplification with one or two EDFAs, uniform gain can be realized over a 75-nm bandwidth covering the C and L bands [94].

A parallel configuration has also been developed for EDFAs capable of amplifying over the C and L bands simultaneously [95]. In this approach, the incoming WDM signal is split into two branches, which amplify the C-band and L-band signals separately using an optimized EDFA in each branch. The two-arm design has produced a relatively uniform gain of 24 dB over a bandwidth as large as 80 nm when pumped with 980-nm semiconductor lasers while maintaining a noise figure of about 6 dB [55]. The two-arm or two-stage amplifiers are complex devices and contain multiple components, such as optical filters and isolators, within them for optimizing the amplifier performance. An alternative approach to broadband EDFAs uses a fluoride fiber in place of silica fibers as the host medium in which erbium ions are doped. Gain flatness over a 76-nm bandwidth has been realized by doping a tellurite fiber with erbium ions [96]. Although such EDFAs are simpler in design compared with multistage amplifiers, they suffer from the splicing difficulties because of the use of nonsilica glasses.

Starting in 2001, high-capacity lightwave systems began to use the short-wavelength region—the so-called S band—extending from 1470 to 1520 nm [97]. Erbium ions cannot provide gain in this spectral band. Thulium-doped fiber amplifiers have been developed for this purpose, and they are capable of providing flat gain in the wavelength range 1480–1510 nm when pumped using 1420-nm and 1560-nm semiconductor lasers [98]. Both lasers are needed to reach the \( 3F_4 \rightarrow 3H_4 \) transition. Raman amplification can also be used for the S band, and such amplifiers were under development in 2001.
6.4.6 Distributed-Gain Amplifiers

Most EDFAs provide 20–25 dB amplification over a length ∼10 m through a relatively high density of dopants (∼500 parts per million). Since such EDFAs compensate for losses accumulated over 80–100 km in a relatively short distance of 10–20 m, they are referred to as the lumped amplifiers. Similar to the case of Raman amplification, fiber losses can also be compensated through distributed amplification. In this approach, the transmission fiber itself is lightly doped (dopant density ∼50 parts per billion) to provide the gain distributed over the entire fiber length such that it compensates for fiber losses locally. Such an approach results in a virtually transparent fiber at a specific wavelength when the fiber is pumped using the bidirectional pumping configuration. The scheme is similar to that discussed in Section 6.3 for distributed Raman amplifiers, except that the dopants provide the gain instead of the nonlinear phenomenon of SRS. Although considerable research has been done on distributed EDFAs [99]–[106], this scheme has not yet been used commercially as it requires special fibers.

Ideally, one would like to compensate for fiber losses in such a way that the signal power does not change at all during propagation. Such a performance is, however, never realized in practice as the pump power is not uniform along the fiber length because of fiber losses at the pump wavelength. Pumping at 980 nm is ruled out because fiber losses exceed 1 dB/km at that wavelength. The optimal pumping wavelength for distributed EDFAs is 1.48 µm, where losses are about 0.25 dB/km. If we include pump absorption by dopants, total pump losses typically exceed 0.4 dB/km, resulting in losses of 10 dB for a fiber length of only 25 km. If the fiber is pumped unidirectionally by injecting the pump beam from one end, nonuniform pumping leads to large variations in the signal power. A bidirectional pumping configuration is therefore used in which the fiber is pumped from both ends by using two 1.48-µm lasers. In general, variations in the signal power due to nonuniform pumping can be kept small for a relatively short fiber length of 10–15 km [99]. For practical reasons, it is important to increase the fiber length close to 50 km or more so that the pumping stations could be spaced that far apart. In a 1995 experiment, 11.5-ps pulses were transmitted over 93.4 km of a distributed EDFA by injecting up to 90 mW of pump power from each end [104]. The signal power was estimated to vary by a factor of more than 10 because of the nonuniform pumping.

The performance of distributed EDFAs depends on the signal wavelength since both the noise figure and the pump power required to achieve transparency change with the signal wavelength [103]. In a 1996 experiment, a 40-Gb/s return-to-zero (RZ) signal was transmitted over 68 km by using 7.8-ps optical pulses [105]. The ASE noise added to the signal is expected to be smaller than lumped EDFAs as the gain is relatively small all along the fiber. Computer simulations show that the use of distributed amplification for nonreturn-to-zero (NRZ) systems has the potential of doubling the pump-station spacing in comparison with the spacing for the lumped amplifiers [106]. For long fiber lengths, one should consider the effect of SRS in distributed EDFAs (pumped at 1.48 μm) because the pump-signal wavelength difference lies within the Raman-gain bandwidth, and the signal experiences not only the gain provided by the dopants but also the gain provided by SRS. The SRS increases the net gain and reduces the noise figure for a given amount of pump power.
6.5 System Applications

Fiber amplifiers have become an integral part of almost all fiber-optic communication systems installed after 1995 because of their excellent amplification characteristics such as low insertion loss, high gain, large bandwidth, low noise, and low crosstalk. In this section we first consider the use of EDFAs as preamplifiers at the receiver end and then focus on the design issues for long-haul systems employing a cascaded chain of optical amplifiers.

6.5.1 Optical Preamplification

Optical amplifiers are routinely used for improving the sensitivity of optical receivers by preamplifying the optical signal before it falls on the photodetector. Preamplification of the optical signal makes it strong enough that thermal noise becomes negligible compared with the noise induced by the preamplifier. As a result, the receiver sensitivity can be improved by 10–20 dB using an EDFA as a preamplifier [107]–[112]. In a 1990 experiment [107], only 152 photons/bit were needed for a lightwave system operating at bit rates in the range 0.6–2.5 Gb/s. In another experiment [110], a receiver sensitivity of $-37.2$ dBm (147 photons/bit) was achieved at the bit rate of 10 Gb/s. It is even possible to use two preamplifiers in series; the receiver sensitivity improved by 18.8 dB with this technique [109]. An experiment in 1992 demonstrated a record sensitivity of $-38.8$ dBm (102 photons/bit) at 10 Gb/s by using two EDFAs [111]. Sensitivity degradation was limited to below 1.2 dB when the signal was transmitted over 45 km of dispersion-shifted fiber.

To calculate the receiver sensitivity, we need to include all sources of current noise at the receiver. The most important performance issue in designing optical preamplifiers is the contamination of the amplified signal by the ASE. Because of the incoherent nature of spontaneous emission, the amplified signal is noisier than the input signal.

Following Sections 4.4.1 and 6.1.3, the photocurrent generated at the detector can be written as

$$I = R\sqrt{GE_s + |E_{sp}|^2 + i_s + iT},$$

(6.5.1)

where $R$ is the photodetector responsivity, $G$ is the amplifier gain, $E_s$ is the signal field, $E_{sp}$ is the optical field associated with the ASE, and $i_s$ and $iT$ are current fluctuations induced by the shot noise and thermal noise, respectively, within the receiver. The average value of the current consists of

$$\bar{I} = R(GP_s + P_{sp}),$$

(6.5.2)

where $P_s = |E_s|^2$ is the optical signal before its preamplification, and $P_{sp}$ is the ASE noise power added to the signal with the magnitude

$$P_{sp} = |E_{sp}|^2 = S_{sp}\Delta\nu_{sp}.$$  

(6.5.3)

The spectral density $S_{sp}$ is given by Eq. (6.1.15) and $\Delta\nu_{sp}$ is the effective bandwidth of spontaneous emission set by the amplifier bandwidth or the filter bandwidth if an optical filter is placed after the amplifier. Notice that $E_{sp}$ in Eq. (6.5.1) includes only
the component of ASE that is copolarized with the signal as the orthogonally polarized component cannot beat with the signal.

The current noise $\Delta I$ consists of fluctuations originating from the shot noise, thermal noise, and ASE noise. The ASE-induced current noise has its origin in the beating of $E_s$ with $E_{sp}$ and the beating of $E_{sp}$ with itself. To understand this beating phenomenon more clearly, notice that the ASE field $E_{sp}$ is broadband and can be written in the form

$$ E_{sp} = \int \sqrt{S_{sp}} \exp(\phi_n - i\omega_n t) d\omega_n, $$  

(6.5.4)

where $\phi_n$ is the phase of the noise-spectral component at the frequency $\omega_n$, and the integral extends over the entire bandwidth of the amplifier (or optical filter). Using $E_s = \sqrt{P_s} \exp(\phi_s - i\omega_{st} t)$, the interference term in Eq. (6.5.1) consists of two parts and leads to current fluctuations of the form

$$ i_{sig-sp} = 2R \int \frac{(GP_sS_{sp})^{1/2}}{\sqrt{2}} \cos \theta_1 d\omega_n, \quad i_{sp-sp} = 2R \int \frac{S_{sp} \cos \theta_2 d\omega_n d\omega'_n}{\sqrt{2}}, $$  

(6.5.5)

where $\theta_1 = (\omega_n - 2\omega_{st}) t + \phi_n - \phi_s$ and $\theta_2 = (\omega_n - \omega_{sp}) t + \phi_{sp}' - \phi_n$ are two rapidly varying random phases. These two contributions to current noise are due to the beating of $E_s$ with $E_{sp}$ and the beating of $E_{sp}$ with itself, respectively. Averaging over the random phases, the total variance $\sigma^2 = \langle (\Delta I)^2 \rangle$ of current fluctuations can be written as [5]

$$ \sigma^2 = \sigma^2_T + \sigma^2_s + \sigma^2_{sig-sp} + \sigma^2_{sp-sp}, $$  

(6.5.6)

where $\sigma^2_T$ is the thermal noise and the remaining three terms are [113]

$$ \sigma^2_s = 2q \frac{\sqrt{R(GP_s + P_{sp})}}{\sqrt{2}} \Delta f, $$  

(6.5.7)

$$ \sigma^2_{sig-sp} = 4R^2 GP_s S_{sp} \Delta f, $$  

(6.5.8)

$$ \sigma^2_{sp-sp} = 4R^2 S_{sp}^2 \Delta V_{opt} \Delta f, $$  

(6.5.9)

where $\Delta V_{opt}$ is the bandwidth of the optical filter and $\Delta f$ is the electrical noise bandwidth of the receiver. The shot-noise term $\sigma^2_s$ is the same as in Section 4.4.1 except that $P_{sp}$ has been added to $GP_s$ to account for the shot noise generated by spontaneous emission.

The BER can be obtained by following the analysis of Section 4.5.1. As before, it is given by

$$ BER = \frac{1}{2} \text{erfc}(Q / \sqrt{2}), $$  

(6.5.10)

with the $Q$ parameter

$$ Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} = \frac{RG(2P_{rec})}{\sigma_1 + \sigma_0}. $$  

(6.5.11)

Equation (6.5.11) is obtained by assuming zero extinction ratio ($I_0 = 0$) so that $I_1 = RGP_1 = RG(2P_{rec})$, where $P_{rec}$ is the receiver sensitivity for a given value of BER ($Q = 6$ for BER = $10^{-9}$). The RMS noise currents $\sigma_1$ and $\sigma_0$ are obtained from Eqs. (6.5.6)–(6.5.9) by setting $P_s = P_1 = 2P_{rec}$ and $P_s = 0$, respectively.

The analysis can be simplified considerably by comparing the magnitude of various terms in Eqs. (6.5.6). For this purpose it is useful to substitute $S_{sp}$ from Eq. (6.1.15),
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Use \( R = \eta q / h \nu \) and Eq. (6.1.19), and write Eqs. (6.5.7)–(6.5.9) in terms of the amplifier noise figure \( F_n \) as

\[
\sigma_s^2 = 2q^2 \eta \bar{G} \Delta f / h \nu, \tag{6.5.12}
\]
\[
\sigma_{\text{sig}-\text{sp}}^2 = 2(q \eta \bar{G})^2 F_n P_s \Delta f / h \nu, \tag{6.5.13}
\]
\[
\sigma_{\text{sp}-\text{sp}}^2 = (q \eta \bar{G} F_n)^2 \Delta \nu_{\text{opt}} \Delta f, \tag{6.5.14}
\]

where the \( \mathcal{R}P_{\text{sp}} \) term was neglected in Eq. (6.5.7) as it contributes negligibly to the shot noise. A comparison of Eqs. (6.5.12) and (6.5.13) shows that \( \sigma_s^2 \) can be neglected in comparison with \( \sigma_{\text{sig}-\text{sp}}^2 \) as it is smaller by a large factor \( \eta \bar{G} F_n \). The thermal noise \( \sigma_T^2 \) can also be neglected in comparison with the dominant terms. The noise currents \( \sigma_1 \) and \( \sigma_0 \) are thus well approximated by

\[
\sigma_1 = (\sigma_{\text{sig}-\text{sp}}^2 + \sigma_{\text{sp}-\text{sp}}^2)^{1/2}, \quad \sigma_0 = \sigma_{\text{sp}-\text{sp}}. \tag{6.5.15}
\]

The receiver sensitivity is obtained by substituting Eq. (6.5.15) in Eq. (6.5.11), using Eqs. (6.5.13) and (6.5.14) with \( P_s = 2 \bar{P}_{\text{rec}} \), and solving for \( \bar{P}_{\text{rec}} \). The result is

\[
\bar{P}_{\text{rec}} = h \nu F_n \Delta f [Q^2 + Q(\Delta \nu_{\text{opt}} / \Delta f)^{1/2}]. \tag{6.5.16}
\]

The receiver sensitivity can also be written in terms of the average number of photons/bit, \( \bar{N}_p \), by using \( \bar{P}_{\text{rec}} = \bar{N}_p h \nu B \). Taking \( \Delta f = B/2 \) as a typical value of the receiver bandwidth, \( \bar{N}_p \) is given by

\[
\bar{N}_p = \frac{1}{2} F_n [Q^2 + Q(2 \Delta \nu_{\text{opt}} / B)^{1/2}]. \tag{6.5.17}
\]

Equation (6.5.17) is a remarkably simple expression for the receiver sensitivity. It shows clearly why amplifiers with a small noise figure must be used; the receiver sensitivity degrades as \( F_n \) increases. It also shows how optical filters can improve the receiver sensitivity by reducing \( \Delta \nu_{\text{opt}} \). Figure 6.20 shows \( \bar{N}_p \) as a function of \( \Delta \nu_{\text{opt}} / B \) for several values of the noise figure \( F_n \) by using \( Q = 6 \), a value required to achieve a BER of \( 10^{-9} \). The minimum optical bandwidth is equal to the bit rate to avoid blocking the signal. The minimum value of \( F_n \) is 2 for an ideal amplifier (see Section 8.1.3). Thus, by using \( Q = 6 \), the best receiver sensitivity from Eq. (6.5.17) is \( \bar{N}_p = 44.5 \) photons/bit. This value should be compared with \( \bar{N}_p = 10 \) for an ideal receiver (see Section 4.5.3) operating in the quantum-noise limit. Of course, \( \bar{N}_p = 10 \) is never realized in practice because of thermal noise; typically, \( \bar{N}_p \) exceeds 1000 for \( p-i-n \) receivers without optical amplifiers. The analysis of this section shows that \( \bar{N}_p < 100 \) can be realized when optical amplifiers are used to preamplify the signal received despite the degradation caused by spontaneous-emission noise. The effect of a finite laser linewidth on the receiver sensitivity has also been included with similar conclusions [114].

Improvements in the receiver sensitivity, realized with an EDFA acting as a preamplifier, can be used to increase the transmission distance of point-to-point fiber links used for intercity and interisland communications. Another EDFA acting as a power booster is often used to increase the launched power to levels as high as 100 mW. In a 1992 experiment, a 2.5-Gb/s signal was transmitted over 318 km by such a technique [115]. Bit rate was later increased to 5 Gb/s in an experiment [116] that used two...
EDFAs to boost the signal power from $-8$ to $15.5$ dBm (about 35 mW). This power level is large enough that SBS becomes a problem. SBS can be suppressed through phase modulation of the optical carrier that broadens the carrier linewidth to 200 MHz or more. Direct modulation of lasers also helps through frequency chirping that broadens the signal spectrum. In a 1996 experiment, a 10-Gb/s signal was transmitted over 442 km using two remotely pumped in-line amplifiers [117].

### 6.5.2 Noise Accumulation in Long-Haul Systems

Optical amplifiers are often cascaded to overcome fiber losses in a long-haul lightwave system. The buildup of amplifier-induced noise is the most critical factor for such systems. There are two reasons behind it. First, in a cascaded chain of optical amplifiers (see Fig. 5.1), the ASE accumulates over many amplifiers and degrades the optical SNR as the number of amplifiers increases [118]–[121]. Second, as the level of ASE grows, it begins to saturate optical amplifiers and reduce the gain of amplifiers located further down the fiber link. The net result is that the signal level drops further while the ASE level increases. Clearly, if the number of amplifiers is large, the SNR will degrade so much at the receiver that the BER will become unacceptable. Numerical simulations show that the system is self-regulating in the sense that the total power obtained by adding the signal and ASE powers remains relatively constant. Figure 6.21 shows this self-regulating behavior for a cascaded chain of 100 amplifiers with 100-km spacing and 35-dB small-signal gain. The power launched by the transmitter is 1 mW. The other parameters are $P_{\text{out}}^*=8$ mW, $n_{\text{op}}=1.3$, and $G_{0}\exp(-\alpha L_A)=3$, where $L_A$ is the

![Figure 6.20](image)

**Figure 6.20**: Receiver sensitivity versus optical-filter bandwidth for several values of the noise figure $F_n$ when an optical amplifier is used for preamplification of the received signal.
amplifier spacing. The signal and ASE powers become comparable after 10,000 km, indicating the SNR problem at the receiver.

To estimate the SNR associated with a long-haul lightwave system, we assume that all amplifiers are spaced apart by a constant distance $L_A$, and the amplifier gain $G \equiv \exp(\alpha L_A)$ is just large enough to compensate for fiber losses in each fiber section. The total ASE power for a chain of $N_A$ amplifiers is then obtained by multiplying Eq. (6.5.3) with $N_A$ and is given by

$$P_{sp} = 2N_A S_{sp} \Delta \nu_{opt} = 2n_{sp} h \nu_0 N_A (G - 1) \Delta \nu_{opt}, \quad (6.5.18)$$

where the factor of 2 accounts for the unpolarized nature of ASE. We can use this equation to find the optical SNR using $SNR_{opt} = P_{in}/P_{sp}$. However, optical SNR is not the quantity that determines the receiver performance. As discussed earlier, the electrical SNR is dominated by the signal-spontaneous beat noise generated at the photodetector. If we include only this dominant contribution, the electrical SNR is related to optical SNR as

$$SNR_{el} = \frac{R^2 P_{in}^2}{N_A G_{sig-sp}^2} = \frac{\Delta \nu_{opt}}{2\Delta f} SNR_{opt} \quad (6.5.19)$$

if we use Eq. (6.5.8) with $G = 1$ assuming no net amplification of the input signal.

We can now evaluate the impact of multiple amplifiers. Clearly, the electrical SNR can become quite small for large values of $G$ and $N_A$. For a fixed system length $L_T$, the number of amplifiers depends on the amplifier spacing $L_A$ and can be reduced by increasing it. However, a longer amplifier spacing will force one to increase the gain of each amplifier since $G = \exp(\alpha L_A)$. Noting that $N_A = L_T/L_A = \alpha L_T/\ln G$, we find that $SNR_{el}$ scales with $G$ as $\ln G/(G - 1)$ and can be increased by lowering the gain of each amplifier. In practice, the amplifier spacing $L_A$ cannot be made too small because...
of cost considerations. To estimate the optimum value of $L_A$, Fig. 6.22 shows the total system length $L_T$ as a function of $L_A$ for several values of input powers $P_{in}$ assuming that an electrical SNR of 20 dB is required for the system to function properly and using $\alpha = 0.2$ dB/km, $n_{sp} = 1.6$ (noise figure 5 dB), and $\Delta f = 10$ GHz. The main point to note is that amplifier spacing becomes smaller as the system length increases. Typically, $L_A$ is kept near 50 km for undersea systems but can be increased to 80 km or so for terrestrial systems with link lengths under 3000 km. Although amplifier spacing can be improved by increasing the input power $P_{in}$, in practice, the maximum power that can be launched is limited by the onset of various nonlinear effects. We turn to this issue next.

### 6.5.3 ASE-Induced Timing Jitter

The amplifier noise can also induce timing jitter in the bit stream by shifting optical pulses from their original time slot in a random fashion. Such jitter was first studied in 1986 in the context of solitons and is called the Gordon–Haus jitter [122]. It was later recognized that timing jitter can occur with any transmission format [NRZ, RZ, or chirped RZ (CRZ)] and imposes a fundamental limitation on all long-haul systems designed with a cascaded chain of optical amplifiers [123]–[126].

The physical origin of ASE-induced jitter can be understood by noting that optical amplifiers affect not only the amplitude but also the phase of the amplified signal as apparent from Eq. (6.5.32) or Eq. (6.5.34). Time-dependent variations in the optical phase lead to a change in the signal frequency by a small amount. Since the group velocity depends on the frequency because of dispersion, the speed at which a soliton propagates through the fiber is affected by each amplifier in a random fashion. Such
random speed changes produce random shifts in the pulse position at the receiver and are responsible for the timing jitter.

Timing jitter induced by the ASE noise can be calculated using the moment method. According to this method, changes in the pulse position $q$ and the frequency $\Omega$ along the link length are calculated using

$$q(z) = \frac{1}{E} \int_{-\infty}^{\infty} t |A(z,t)|^2 dt,$$

(6.5.20)

$$\Omega(z) = \frac{i}{2E} \int_{-\infty}^{\infty} \left( A^* \frac{\partial A}{\partial t} - A \frac{\partial A^*}{\partial t} \right) dt,$$

(6.5.21)

where $E = \int_{-\infty}^{\infty} |A|^2 dt$ represents the pulse energy.

The NLS equation can be used to find how $T$ and $W$ evolve along the fiber link. Differentiating Eqs. (6.5.20) and (6.5.21) with respect to $z$ and using Eq. (6.5.31), we obtain

$$\frac{d\Omega}{dz} = \sum_i \delta\Omega_i \delta(z - z_i),$$

(6.5.22)

$$\frac{dq}{dz} = \beta_2 \Omega + \sum_i \delta q_i \delta(z - z_i),$$

(6.5.23)

where $\delta\Omega_i$ and $\delta q_i$ are the random frequency and position changes imparted by noise at the $i$th amplifier and the sum is over the total number $N_A$ of amplifiers. These equations show that frequency fluctuations induced by an amplifier become temporal fluctuations because of GVD; no jitter occurs when $\beta_2 = 0$.

Equations (6.5.22) and (6.5.23) can be integrated in a straightforward manner. For a cascaded chain of $N_A$ amplifiers with spacing $L_A$, the pulse position at the last amplifier is given by

$$q_f = \sum_{n=1}^{N_A} \delta q_n + \bar{\beta}_2 L_A \sum_{n=1}^{N_A} \sum_{i=1}^{n-1} \delta\Omega_i,$$

(6.5.24)

where $\bar{\beta}_2$ is the average value of the GVD. Timing jitter is calculated from this equation using $\sigma_T^2 = \langle q_f^2 \rangle - \langle q_f \rangle^2$ together with $\langle q_f \rangle = 0$. The average can be performed by noting that fluctuations at two different amplifiers are not correlated. However, the timing jitter depends not only on the variances of position and frequency fluctuations but also on the cross-correlation function $\langle \delta q \delta\Omega \rangle$ at the same amplifier. These quantities depend on the pulse amplitude $A(z_i,t)$ at the amplifier location $z_i$ (see Section 9.5).

Consider a low-power lightwave system employing the CRZ format and assume that the input pulse is in the form of a chirped Gaussian pulse. As seen in Section 2.4, the pulse maintains its Gaussian shape on propagation such that

$$A(z,t) = a \exp[i\phi - i\Omega(t-q) - (1 + iC)(t-q)^2/2T^2],$$

(6.5.25)

where the amplitude $a$, phase $\phi$, frequency $\Omega$, position $q$, chirp $C$, and width $T$ all are functions of $z$. The variances and cross-correlation of $\delta q$ and $\delta\Omega$ at the location of the $i$th amplifier are found to be

$$\langle (\delta\Omega)^2 \rangle = \frac{(S_{\Omega p}/E_0)}{(1 + C_i^2/T_i^2)},$$

(6.5.26)
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Figure 6.23: ASE-induced timing jitter as a function of system length for several values of the average dispersion $\bar{\beta}_2$.

$$\left\langle (\delta q)^2 \right\rangle = \left( \frac{S_{sp}}{E_0} \right) T_i^2$$
$$\left\langle \delta \Omega \delta q \right\rangle = \left( \frac{S_{sp}}{E_0} \right) C_i,$$

where $E_0$ is the input pulse energy and $C_i$ and $T_i$ are the chirp and width at $z = z_i$. These quantities can be calculated easily using the theory of Section 2.4. Note that the ratio $(1 + C_i^2)/T_i^2$ is related to the spectral width that does not change if the nonlinear effects are negligible. It can be replaced by $T_m^{-2}$, where $T_m$ is the minimum width occurring when the pulse is unchirped.

Many lightwave systems employ the postcompensation technique in which a fiber is placed at the end of the last amplifier to reduce the net accumulated dispersion (see Section 7.4). Using Eqs. (6.5.24)–(6.5.27), the timing jitter for a CRZ system employing postcompensation is found to be [126]

$$\sigma_i^2 = \left( \frac{S_{sp}}{E_0} \right) T_m^2 \left[ N_A + N_A (N_A d + C_0 + d_f)^2 \right],$$

where $C_0$ is the input chirp, $d = \beta_2 L_A / T_m^2$, and $d_f = \beta_{2f} L_f / T_m^2$ for a postcompensation fiber of length $L_f$ and dispersion $\beta_{2f}$. Several points are noteworthy. First, if postcompensation is not used ($d_f = 0$), the dominant term in Eq. (6.5.28) varies as $N_A^3 d^2$. This is the general feature of the ASE jitter resulting from frequency fluctuations [122]. Second, if the average dispersion of the fiber link is zero, the cubic term vanishes, and the jitter increases only linearly with $N_A$. Third, the smallest value of the jitter occurs when $N_A d + C_0 + d_f = 0$. This condition corresponds to zero net dispersion over the entire link, including the fiber used to chirp the pulse initially.

The average dispersion of the fiber link can lead to considerable timing jitter in CRZ systems when postcompensation is not used. Figure 6.23 shows the timing jitter as a function of the total system length $L_T = N_A L_A$ for a 10-Gb/s system using four values of $\beta_2$ with $T_m = 30$ ps, $L_A = 50$ km, $C_0 = 0.2$, and $S_{sp}/E_0 = 10^{-4}$. The ASE-induced jitter becomes a significant fraction of the pulse width for values of $|\bar{\beta}_2|$ as small as 0.2 ps$^2$/km because of the cubic dependence of $\sigma_i^2$ on the system length $L_T$. Such jitter would lead to large power penalties, as discussed in Section 4.6.3, if left
uncontrolled. The tolerable value of the jitter can be estimated assuming the Gaussian statistics for $q$ so that

$$p(q) = (2\pi \sigma_t^2)^{-1/2} \exp\left(-q^2 / 2\sigma_t^2\right). \quad (6.5.29)$$

The BER can be calculated following the method of Section 4.5. If we assume that an error occurs whenever the pulse has moved out of the bit slot, we need to find the accumulated probability for $|q|$ to exceed $T_B/2$, where $T_B \equiv 1/B$ is the bit slot. This probability is found to be

$$\text{BER} = 2 \int_{T_B/2}^\infty p(q) dq = \text{erfc} \left( \frac{T_B}{2\sqrt{2}\sigma_t} \right) \approx \frac{4\sigma_t}{\sqrt{2\pi}T_B} \exp\left(-\frac{T_B^2}{8\sigma_t^2}\right), \quad (6.5.30)$$

where $\text{erfc}$ stands for the complimentary error function defined in Eq. (4.5.5). To reduce the BER below $10^{-9}$ for $\sigma_t/T_B$ should be less than 8% of the bit slot, resulting in a tolerable value of the jitter of 8 ps for 10-Gb/s systems and only 2 ps for 40-Gb/s systems. Clearly, the average dispersion of a fiber link should nearly vanish if the system is designed not to be limited by the ASE-induced jitter. This can be accomplished through dispersion management discussed in Chapter 7.

### 6.5.4 Accumulated Dispersive and Nonlinear Effects

Many single-channel experiments performed during the early 1990s demonstrated the benefits of in-line amplifiers for increasing the transmission distance of point-to-point fiber links [127]–[132]. These experiments showed that fiber dispersion becomes the limiting factor in periodically amplified long-haul systems. Indeed, the experiments were possible only because the system was operated close to the zero-dispersion wavelength of the fiber link. Moreover, the zero-dispersion wavelength varied along the link in such a way that the total dispersion over the entire link length was quite small at the operating wavelength of 1.55 µm. By 1992, the total system length could be increased to beyond 10,000 km using such dispersion-management techniques. In a 1992 experiment [130], a 2.5-Gb/s signal was transmitted over 10,073 km using 199 EDFAs. An effective transmission distance of 21,000 km at 2.5 Gb/s and of 14,300 km at 5 Gb/s was demonstrated using a recirculating fiber loop [133].

A crude estimate of dispersion-limited $L_T$ can be obtained if the input power is low enough that one can neglect the nonlinear effects during signal transmission. Since amplifiers compensate only for fiber losses, dispersion limitations discussed in Section 5.2.2 and shown in Fig. 5.4 apply for each channel of a WDM system if $L$ is replaced by $L_T$. From Eq. (5.2.3), the dispersion limit for systems making use of standard fibers ($\beta_2 \approx -20 \text{ ps}^2/\text{km} \text{ at } 1.55 \mu\text{m}$) is $B^2L_T < 3000 \text{ (Gb/s)}^2\cdot\text{km}$: The distance is limited to below 30 km at 10 Gb/s for such fibers. An increase by a factor of 20 can be realized by using dispersion-shifted fibers. To extend the distance to beyond 5000 km at 10 Gb/s, the average GVD along the link should be smaller than $\bar{\beta}_2 = -0.1 \text{ ps}^2/\text{km}$.

The preceding estimate is crude since it does not include the impact of the nonlinear effects. Even though power levels are relatively modest for each channel, the nonlinear effects can become quite important because of their accumulation over long distances [29]. Moreover, amplifier noise often forces one to increase the channel
power to more than 1 mW in order to maintain a high SNR (or a high $Q$ factor). The accumulation of the nonlinear effects then limits the system length $L_T$ [134]–[147]. For single-channel systems, the most dominant nonlinear phenomenon that limits the system performance is self-phase modulation (SPM). An estimate of the power limitation imposed by the SPM can be obtained from Eq. (2.6.15). In general, the condition $\phi_{NL} \ll 1$ limits the total link length to $L_T \ll L_{NL}$, where the nonlinear length is defined as $L_{NL} = \left( \gamma P \right)^{-1}$. Typically, $\gamma \sim 1 \text{ W}^{-1}/\text{km}$, and the link length is limited to below 1000 km even for $P = 1 \text{ mW}$.

The estimate of the SPM-limited distance is too simplistic to be accurate since it completely ignores the role of fiber dispersion. In fact, since the dispersive and nonlinear effects act on the optical signal simultaneously, their mutual interplay becomes quite important. As discussed in Section 5.3, it is necessary to solve the nonlinear Schrödinger equation

$$\frac{\partial A}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} = i\gamma |A|^2 A - \frac{\alpha}{2} A$$

(6.5.31)

numerically, while including the gain and ASE noise at the location of each amplifier. Such an approach is indeed used to quantify the impact of nonlinear effects on the performance of periodically amplified lightwave systems [134]–[148]. A common technique solves Eq. (6.5.31) in each fiber segment using the split-step Fourier method [56]. At each optical amplifier, the noise is added using

$$A_{\text{out}}(t) = \sqrt{G} A_{\text{in}}(t) + a_n(t),$$

(6.5.32)

where $G$ is the amplification factor. The spontaneous-emission noise field $a_n$ added by the amplifier vanishes on average but its second moment satisfies

$$\langle a_n(t) a_n(t') \rangle = S_{sp} \delta(t - t'),$$

(6.5.33)

where the noise spectral density $S_{sp}$ is given by Eq. (6.1.15).

In practice, Eq. (6.5.32) is often implemented in the frequency domain as

$$\tilde{A}_{\text{out}}(\nu) = \sqrt{G} \tilde{A}_{\text{in}}(\nu) + \tilde{a}_n(\nu),$$

(6.5.34)

where a tilde represents the Fourier transform. The noise $\tilde{a}_n(\nu)$ is assumed to be frequency independent (white noise) over the whole amplifier bandwidth, or the filter bandwidth if an optical filter is used after each amplifier. Mathematically, $\tilde{a}_n(\nu)$ is a complex Gaussian random variable whose real and imaginary parts have the spectral density $S_{sp}/2$. The system performance is quantified through the $Q$ factor as defined in Eq. (4.5.10) and related directly to the BER through Eq. (4.5.9).

As an example of the numerical results, the curve (a) in Fig. 6.24 shows variations of the $Q$ factor with the average input power for a NRZ, single-channel lightwave system designed to operate at 5 Gb/s over 9000 km of constant-dispersion fibers [$D = 1 \text{ ps/(km-nm)}$] with 40-km amplifier spacing [134]. Since $Q < 6$ for all input powers, such a system cannot operate reliably in the absence of in-line filters ($Q > 6$ is required for a BER of $< 10^{-9}$). An optical filter of 150-GHz bandwidth, inserted after every amplifier, reduces the ASE for the curve (b). In the presence of optical filters, $Q > 6$ can be realized only at a specific value of the average input power (about 0.5 mW). This
behavior can be understood by noting that as the input power increases, the system performance improves initially because of a better SNR but becomes worse at high input powers as the nonlinear effects (SPM) begins to dominate.

The role of dispersion can be minimized either by operating close to the zero-dispersion wavelength of the fiber or by using a dispersion management technique in which the fiber GVD alternates its sign in such a way that the average dispersion is close to zero (see Chapter 7). In both cases, the GVD parameter $\beta_2$ fluctuates because of unintentional variations in the zero-dispersion wavelength of various fiber segments. The curve (c) in Fig. 6.24 is drawn for a 6-Gb/s system for the case of a Gaussian distribution of $\beta_2$ with a standard deviation of 0.3 ps$^2$/km. The filter bandwidth is taken to be 60 GHz [134]. The curve (d) shows the dispersion-managed case for a 10-Gb/s system with a filter bandwidth of 50 GHz. All other parameters remain the same. Clearly, system performance can improve considerably with dispersion management, although the input pump power needs to be optimized in each case.

### 6.5.5 WDM-Related Impairments

The advantages of EDFAs for WDM systems were demonstrated as early as 1990 [149]–[154]. In a 1993 experiment, four channels were transmitted over 1500 km using 22 cascaded amplifiers [150]. By 1996, 55 channels, spaced apart by 0.8 nm and each operating at 20 Gb/s, were transmitted over 150 km by using two in-line amplifiers, resulting in a total bit rate of 1.1 Tb/s and the $BL$ product of 165 (Tb/s)-km [151]. For submarine applications, one needs to transmit a large number of channels over a distance of more than 5000 km. Such systems employ a large number of cascaded amplifiers and are affected most severely by the amplifier noise. Already in 1996, transmission at 100 Gb/s (20 channels at 5 Gb/s) over a distance of 9100 km was possible using the polarization-scrambling and forward-error correction techniques [152]. By 2001, transmission at 2.4 Tb/s (120 channels at 20 Gb/s) over 6200 km has been realized within the C band using EDFAs every 50 km [154]. The adjacent channels
were orthogonally polarized for reducing the nonlinear effects resulting from a relatively small channel spacing of 42 GHz. This technique is referred to as polarization multiplexing and is quite useful for WDM systems.

The two major nonlinear phenomena affecting the performance of WDM systems are the cross-phase modulation (XPM) and four-wave mixing (FWM). FWM can be avoided by using dispersion management such that the GVD is locally high all along the fiber but quite small on average. The SPM and XPM then become the most limiting factors for WDM systems. The XPM effects within an EDFA are normally negligible because of a small length of doped fiber used. The situation changes for the L-band amplifiers, which operate in the 1570- to 1610-nm wavelength region and require fiber lengths in excess of 100 m. The effective core area of doped fibers used in such amplifiers is relatively small, resulting in larger values of the nonlinear parameter $\gamma$ and enhanced XPM-induced phase shifts. As a result, the XPM can lead to considerable power fluctuations within an L-band amplifier [155]–[160]. A new feature is that such XPM effects are independent of the channel spacing and can occur over the entire bandwidth of the amplifier [156]. The reason for this behavior is that all XPM effects occur before pulses walk off because of group-velocity mismatch. The effects of FWM are also enhanced in L-band amplifiers because of their long lengths [161].

Problems

6.1 The Lorentzian gain profile of an optical amplifier has a FWHM of 1 THz. Calculate the amplifier bandwidths when it is operated to provide 20- and 30-dB gain. Neglect gain saturation.

6.2 An optical amplifier can amplify a 1-µW signal to the 1-mW level. What is the output power when a 1-mW signal is incident on the same amplifier? Assume that the saturation power is 10 mW.

6.3 Explain the concept of noise figure for an optical amplifier. Why does the SNR of the amplified signal degrade by 3 dB even for an ideal amplifier?

6.4 A 250-µm-long semiconductor laser is used as an FP amplifier by biasing it below threshold. Calculate the amplifier bandwidth by assuming 32% reflectivity for both facets and 30-dB peak gain. The group index $n_g = 4$. How much does the bandwidth change when both facets are coated to reduce the facet reflectivities to 1%?

6.5 Complete the derivation of Eq. (6.2.3) starting from Eq. (6.2.1). What should be the facet reflectivities to ensure traveling-wave operation of a semiconductor optical amplifier designed to provide 20-dB gain. Assume that $R_1 = 2R_2$.

6.6 A semiconductor optical amplifier is used to amplify two channels separated by 1 GHz. Each channel can be amplified by 30 dB in isolation. What are the channel gains when both channels are amplified simultaneously? Assume that $P_{in}/P_s = 10^{-3}$, $\tau_c = 0.5$ ns, and $\beta_c = 5$.

6.7 Integrate Eq. (6.2.19) to obtain the time-dependent saturated gain given by Eq. (6.2.20). Plot $G(\tau)$ for a 10-ps square pulse using $G_0 = 30$ dB and $E_s = 10$ pJ.
6.8 Explain why semiconductor optical amplifiers impose a chirp on the pulse during amplification. Derive an expression for the imposed chirp when a Gaussian pulse is incident on the amplifier. Use Eq. (6.2.21) with \( m = 1 \) for the input pulse.

6.9 Discuss the origin of gain saturation in fiber Raman amplifiers. Solve Eqs. (6.3.2) and (6.3.3) with \( \alpha_s = \alpha_p \) and derive Eq. (6.3.8) for the saturated gain.

6.10 A Raman amplifier is pumped in the backward direction using 1 W of power. Find the output power when a 1-\( \mu \)W signal is injected into the 5-km-long amplifier. Assume losses of 0.2 and 0.25 dB/km at the signal and pump wavelengths, respectively, \( A_{\text{eff}} = 50 \, \mu \text{m}^2 \), and \( g_R = 6 \times 10^{-14} \, \text{m/W} \). Neglect gain saturation.

6.11 Explain the gain mechanism in EDFAs. Use Eqs. (6.4.2) and (6.4.3) to derive an expression for the small-signal gain in the steady state.

6.12 Discuss how EDFAs can be used to provide gain in the L band. How can you use them to provide amplification over the both C and L bands?

6.13 Starting from Eq. (6.5.11), derive Eq. (6.5.16) for the sensitivity of a direct-detection receiver when an EDFA is used as a preamplifier.

6.14 Calculate the receiver sensitivity at a BER of \( 10^{-9} \) and \( 10^{-12} \) by using Eq. (6.5.16). Assume that the receiver operates at 1.55 \( \mu \)m with 3-GHz bandwidth. The preamplifier has a noise figure of 4 dB, and a 1-nm optical filter is installed between the preamplifier and the detector.

6.15 Calculate the optical SNR at the output end of a 4000-km lightwave system designed using 50 EDFAs with 4.5-dB noise figure. Assume a fiber-cable loss of 0.25 dB/km at 1.55 \( \mu \)m. A 2-nm-bandwidth optical filter is inserted after every amplifier to reduce the noise.

6.16 Find the electrical SNR for the system of the preceding problem for a receiver of 8-GHz bandwidth.

6.17 Why does ASE induce timing jitter in lightwave systems? How would you design the system to reduce the jitter?

6.18 Use the moment method to find an expression for the timing jitter for a lightwave system employing the CRZ format.

References

REFERENCES

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